Week 2 Wednesday

You Choose

Make sure you know your neighbors' names. Then choose one of the following to discuss briefly:

- If you could live anywhere in the world, where would you choose to live, and why?
- What is the dimension of the quotient ring $\mathbb{R}[x]/\langle x^3 + x + 1\rangle$ as a vector space over $\mathbb{R}$?

Multivariable Polynomials

1. (A) True or (B) False? The quotient ring $k[x_1, \ldots, x_n]/I$ is noetherian for any ideal $I \subseteq k[x_1, \ldots, x_n]$.

2. Let R be a ring. Prove that every ideal of R is finitely generated if and only if, for every ascending chain of ideals

$$I_0 \subseteq I_1 \subseteq I_2 \subseteq \cdots$$

in R, there exists an integer n such that $I_n = I_{n+1} = I_{n+2} = \cdots$.

Let R be the subring of $k[x, y]$ consisting of polynomials in which every term divisible by y is also divisible by x. So, for example, the polynomial $1 + 2xy - xy^2$ is in R, but $2 + 2y + y^2$ is not. (You can take for granted that this is in fact a subring.)

3. (A) True or (B) False? R is noetherian.