Name:

Quiz 4

Part I (10 points). You will get 1 point for each correct answer, 0 points for each blank answer, and -1 point for each incorrect answer. The minimum possible score for this section is 0.

(1) $\mathcal{L}(\mathcal{P}_2(\mathbf{F}), \mathcal{P}_3(\mathbf{F}))$ is isomorphic to $\mathbf{F}^4 \times \mathcal{P}_2(\mathbf{F})'$. T F

(2) Let $U = \{p \in \mathcal{P}_4(\mathbf{R}) : p(1) = p(3) = 0\}$. Then $\mathcal{P}_4(\mathbf{R})/U$ is isomorphic to $\mathbf{R}^4$. T F

(3) If $T \in \mathcal{L}(\mathbf{F}^4)$ is surjective, then $\mathbf{F}^4/\operatorname{null} T$ is isomorphic to $\mathcal{P}_2(\mathbf{F})$. T F

(4) Let $U = \{p \in \mathcal{P}_2(\mathbf{R}) : p(1) = p'(1) = 0\}$. Then $x^2 + U = 1 + U$. T F

(5) Suppose $T \in \mathcal{L}(\mathbf{F}^3, \mathbf{F}^2)$ and T' is injective. Then $\dim(\mathbf{F}^2/\operatorname{range} T) = 2$. T F

(6) Suppose $T \in \mathcal{L}(\mathcal{P}_4(\mathbf{F}))$ and $\dim \operatorname{null} T' = 1$. Then $\dim \operatorname{range} T = 1$. T F

(7) Suppose $T \in \mathcal{L}(V, W)$ and we choose bases for V and W such that $M(T)$ is a 3×5 matrix whose rows span a 3 dimensional space. Then T is injective. T F

(8) Let $\varphi \in \mathcal{P}_3(\mathbf{R})'$ be given by T F

$$\varphi(p) = \int_{-1}^{1} p(x)\, dx.$$

Then $\{p \in \mathcal{P}_3(\mathbf{R}) : \varphi \in \operatorname{span}(p)^0\} = \operatorname{span}(x, x^3)$.

(9) Suppose that U is a subspace of $\mathbf{F}^2$ such that $\mathbf{F}^2 = \operatorname{span}((1,0)) \oplus U$, and let φ_1, φ_2 be dual to the standard basis $(1,0), (0,1)$ of $\mathbf{F}^2$. Then $U^0 = \operatorname{span}(\varphi_2)$. T F

(10) If $T \in \mathcal{L}(V, W)$ and U is a subspace of $\operatorname{null} T$, then the map $\tilde{T} : V/U \to W$ given by $\tilde{T}(v + U) = T(v)$ is a well-defined injective linear map. T F

Part II (10 points).

(11) If V and W are arbitrary vector spaces, show that $V' \times W'$ is isomorphic to $(V \times W)'$.

(12) Let $\varphi_1, \varphi_2, \varphi_3 \in \mathcal{P}_2(\mathbf{R})'$ be defined by $\varphi_1(p) = p(3)$, $\varphi_2(p) = p(-2)$, and $\varphi_3(p) = p(7)$. Is there a basis p_1, p_2, p_3 of $\mathcal{P}_2(\mathbf{R})$ whose dual basis is $\varphi_1, \varphi_2, \varphi_3$? If so, find such a basis and prove that it is actually a basis. Otherwise, explain why no such basis exists.