

Name:

QUIZ 2

Part I (10 points). You will get 1 point for each correct answer, 0 points for each blank answer, and -1 point for each incorrect answer. The minimum possible score for this section is 0.

- (1) Every list of 5 polynomials in $\mathcal{P}_3(\mathbf{F})$ is linearly dependent. T F
- (2) Every basis for $\mathcal{P}_3(\mathbf{F})$ contains a degree 2 polynomial. T F
- (3) Suppose u, v, w is a basis for a vector space V and $w' \in V$. Then u, v, w' is a basis for V if and only if w' is a nonzero scalar multiple of w . T F
- (4) If u, v, w and u', v', w' are both bases for a 3 dimensional vector space V , then $u + u', v + v', w + w'$ is a basis. T F
- (5) Suppose p_0, p_1, p_2, p_3 is a list of four polynomials in $\mathcal{P}_3(\mathbf{R})$ with the property that $p_i(3) = 0$ for each i . Then the list p_0, p_1, p_2, p_3 is linearly dependent. T F
- (6) If v_1, v_2, v_3, v_4 is a basis for V and U is a subspace of V such that $v_1, v_2 \in U$ but $v_3, v_4 \notin U$, then v_1, v_2 is a basis of U . T F
- (7) If U_1, U_2 and U_3 are subspaces of a finite dimensional vector space V , then $\dim(U_1 + U_2 + U_3) = \dim U_1 + \dim U_2 + \dim U_3$. T F
- (8) If U_1, U_2 and W are subspaces of a vector space V such that $U_1 \oplus W = V$ and $U_2 \oplus W = V$, then $U_1 = U_2$. T F
- (9) If v_1, \dots, v_m is linearly independent and $w \in V$ is a vector such that $v_1 + w, \dots, v_m + w$ is linearly dependent, then $w \in \text{span}(v_1, \dots, v_m)$. T F
- (10) If V is a 3 dimensional vector space over \mathbf{C} , then it is 6 dimensional when it is regarded as a vector space over \mathbf{R} . T F

Part II (10 points). Compute dimensions of each of the following vector spaces. Justify your answers with complete proofs.

(11) $V = \{(x, y, z) \in \mathbf{R}^3 : x - 2y - 3z = 0\}$.

(12) $V = \left\{ p \in \mathcal{P}_3(\mathbf{R}) : \int_{-2}^2 p(x) dx = 0 \right\}$.