1. True or False?

If $T \in \mathcal{L}(\mathbf{C}^3)$ is normal, $T(1,1,1) = (2,2,2)$, and $(z_1, z_2, z_3) \in \operatorname{null} T$, then $z_1 + z_2 + z_3 = 0$.

2. True or False?

If V is a finite dimensional inner product space, $T \in \mathcal{L}(V)$ is normal, and

$$\|v\| = \|w\| = 2 \text{ and } Tv = 3v \text{ and } Tw = 4w,$$

then $\|T(v + w)\| = 10$.

3. True or False?

Let V be a finite dimensional real inner product space with $\dim V \geq 2$ and

$$U = \{T \in \mathcal{L}(V) : T \text{ is self-adjoint}\}.$$

Then U is a subspace of $\mathcal{L}(V)$.

3. True or False?

Let V be a finite dimensional real inner product space with $\dim V \geq 2$ and

$$U = \{T \in \mathcal{L}(V) : T \text{ is self-adjoint}\}.$$

Then U is a subspace of $\mathcal{L}(V)$.

What if V is a finite dimensional *complex* inner product space?

4. True or False?

Let V be a finite dimensional inner product space with $\dim V \geq 2$ and

$$U = \{T \in \mathcal{L}(V) : T \text{ is normal}\}.$$

Then U is a subspace of $\mathcal{L}(V)$.

5. Suppose V is a finite dimensional inner product space, $T \in \mathcal{L}(V)$ and U is a subspace of V. Here are two statements.

I. U is invariant under T if and only if its orthogonal complement $U^\perp$ is invariant under the adjoint T^*.

II. U is invariant under T if and only if its annihilator U^0 is invariant under the dual map T'.

Which of these statements is true?

(A) Only I.

(B) Only II.

(C) Both I and II.

(D) Neither of them.