1. Suppose $v_1, \ldots, v_n$ is a linearly *dependent* list in an inner product space V. What happens when I apply the Gram-Schmidt procedure to this list?

(A) I end up with a linearly dependent orthonormal list of vectors $e_1, \ldots, e_m$.

(B) At some step j, I have to divide by 0 to compute e_j.

(C) Neither of the above.

2. True or False?

Let $e_1, \ldots, e_m$ be an orthonormal list in a finite dimensional inner product space V and define $P \in \mathcal{L}(V)$ by

$$P(v) = \langle v, e_1 \rangle e_1 + \cdots + \langle v, e_m \rangle e_m$$

for all $v \in V$. Then P is diagonalizable.

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What is the characteristic polynomial of P?
What is the minimal polynomial of P?

3. True or False?

Suppose V is a finite dimensional complex inner product space and $P \in \mathcal{L}(V)$ is an arbitrary linear operator such that $P^2 = P$. Then there exists an orthonormal list $e_1, \ldots, e_m$ in V such that

$$P(v) = \langle v, e_1 \rangle e_1 + \cdots + \langle v, e_m \rangle e_m$$

for all $v \in V$.

4. True or False?

Suppose $v_1, \ldots, v_m$ is a linearly independent list in an inner product space V. Then there exists $w \in V$ such that

$$\langle v_i, w \rangle > 0$$

for all $i = 1, \ldots, m$.