

1. True or False?

The formula

$$\langle (x_1, x_2), (y_1, y_2) \rangle = |(x_1 + x_2)(y_2 + y_2)|$$

defines an inner product on \mathbf{R}^2 .

2. True or False?

The formula

$$\langle (x_1, x_2), (y_1, y_2) \rangle = 2x_1y_1 + 7x_2y_2$$

defines an inner product on \mathbf{R}^2 .

3. True or False?

The formula

$$\langle (x_1, x_2), (y_1, y_2) \rangle = -3x_1y_1 + x_2y_2$$

defines an inner product on \mathbf{R}^2 .

4. True or False?

The formula

$$\langle (x_1, x_2), (y_1, y_2) \rangle = x_1 y_1$$

defines an inner product on \mathbf{R}^2 .

5. True or False?

If V is a finite dimensional inner product space and $T \in \mathcal{L}(V)$ is an operator such that $\|Tv\| \leq \|v\|$ for all $v \in V$, then $T - \sqrt{2}I$ is invertible.

6. True or False?

Suppose $V = \mathbf{R}^2$ is regarded as an inner product space with the inner product

$$\langle (x_1, x_2), (y_1, y_2) \rangle = 2x_1y_1 + 7x_2y_2.$$

Then the set of vectors orthogonal to $(1, 0)$ is $\{(0, y) : y \in \mathbf{R}\}$.