1. True or False?

If $T \in \mathcal{L}(\mathbf{C}^4)$ and $\operatorname{null} T^3 \neq \operatorname{null} T^4$, then T is nilpotent.

2. True or False?

Suppose T is an operator on a finite dimensional complex vector space V and

$$\begin{aligned}
\dim \operatorname{null}(T - 3I) &= 3 \\
\dim \operatorname{null}(T - 3I)^2 &= 6 \\
\dim \operatorname{null}(T - 3I)^3 &= 8 \\
\dim \operatorname{null}(T - 3I)^4 &= 9 \\
\dim \operatorname{null}(T - 3I)^5 &= 9
\end{aligned}$$

Then the Jordan form of T has 3 Jordan blocks with eigenvalue 3.

3. True or False?

If $T \in \mathcal{L}(\mathbf{C}^3)$ has eigenvalues 0, 1 and -1, then $T^3 = T$.

4. True or False?

If V is a finite dimensional complex vector space and $T \in \mathcal{L}(V)$ is diagonalizable, then $p_{\mathrm{char}} = p_{\mathrm{min}}$.

5. True or False?

Suppose $T \in \mathcal{L}(\mathbf{C}^4)$ is nilpotent and $p_{\mathrm{char}} = p_{\mathrm{min}}$. Then there exists a vector v such that T^3v, T^2v, Tv, v is a basis for $\mathbf{C}^4$.

6. True or False?

If V is an finite dimensional complex vector space and $T \in \mathcal{L}(V)$, then $T^{\dim V}$ is diagonalizable.

7. True or False?

There exists an operator $T \in \mathcal{L}(\mathbf{C}^6)$ such that $p_{\mathrm{char}}(z) = (z-3)^4(z-4)^2$ and $p_{\mathrm{min}}(z) = (z-3)^3(z-4)^2$.

8. True or False?

Suppose V is a finite dimensional complex vector space with $\dim V \geq 2$. The set

$$U = \{T \in \mathcal{L}(V) : T \text{ is nilpotent}\}$$

is a subspace of $\mathcal{L}(V)$.