1. True or False?

Suppose $S : U \to V$ and $T : V \to W$ are injective linear maps. Then TS is also injective.

2. True or False?

Let $T : \mathbf{R}^2 \to \mathbf{R}^3$ be the map $T(x, y) = (2x, 2y, 0)$. Then there exists a basis for $\mathbf{R}^2$ and a basis for $\mathbf{R}^3$ such that the matrix representing T with respect to these bases is

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}.$$

3. Consider the linear map $T : \mathcal{P}_3(\mathbf{R}) \to \mathcal{P}_4(\mathbf{R})$ given by

$$T(f)(z) = z^2 f'(z).$$

(A) T is both injective and surjective.

(B) T is injective but not surjective.

(C) T is surjective but not injective.

(D) T is neither surjective nor injective.

4. True or False?

There exists a surjective linear map $T : \mathcal{P}_2(\mathbf{R}) \to V$ where

$$V = \{p \in \mathcal{P}_4(\mathbf{R}) : p(4) = 0\}.$$

5. True or False?

Suppose V is finite dimensional. Then there exists $T \in \mathcal{L}(V, V)$ such that $\mathrm{range}\, T = \mathrm{null}\, T$.

6. True or False?

Suppose V and W are finite dimensional vector spaces and $T \in \mathcal{L}(V, W)$ is such that $\dim \operatorname{null} T = 2$. Then there exists a basis $v_1, \ldots, v_n$ of V such that, for any basis whatsoever of W, the matrix of T has every entry in the first two columns equal to 0.