1. True or False?

If V is a 1 dimensional vector space and $S, T \in \mathcal{L}(V, V)$, then

$$ST = TS.$$

2. Let V be the subspace inside $\mathbf{R}^{\mathbf{R}}$ of all infinitely differentiable functions. Let $T \in \mathcal{L}(V, V)$ be given by

$$T(f) = f' - f$$

for all $f \in V$.

(A) $\dim \operatorname{null} T = 0$.

(B) $\dim \operatorname{null} T = 1$.

(C) $\operatorname{null} T$ is infinite dimensional.

3. True or False?

The set

$$U = \{T \in \mathcal{L}(\mathbf{R}^5, \mathbf{R}^4) : \dim \operatorname{null} T \geq 3\}$$

is a subspace of $\mathcal{L}(\mathbf{R}^5, \mathbf{R}^4)$.

4. Let V be a finite dimensional vector space and let

$$U = \{T \in \mathcal{L}(V, V) : T \text{ is not injective}\}.$$

(A) U is never a subspace of $\mathcal{L}(V, V)$.

(B) U is sometimes a subspace of $\mathcal{L}(V, V)$.

(C) U is always a subspace of $\mathcal{L}(V, V)$.

5. True or False?

There exists $T \in \mathcal{L}(\mathbf{R}^5, \mathbf{R}^2)$ whose null space equals

$$\{(x_1, x_2, x_3, x_4, x_5) \in \mathbf{R}^5 : x_1 = 3x_2 \text{ and } x_3 = x_4 = x_5\}.$$