Week 10 Thursday

Final Exam

Next Thursday at 8am. Here.

You are allowed to bring *one two-sided handwritten sheet of notes*. No other resources. See website for more information:

https://sagrawalx.github.io/teaching/sp24_math187a/quiz#final

Review

1. Which of the following is *not* a quadratic residue mod 13?

(A) 3

(B) 4

(C) 5

(D) 9

2. What is the order of 5 mod 11?

(A) 1

(B) 2

(C) 5

(D) 10

Follow-up. Would it make sense to include "(E) None of the above" as an option above?

3. Three dice are rolled simultaneously, but two of them fall off the table. The one that landed on the table landed on 6. What is the probability that, when you look under the table, you'll find that the sum of the values on all three dice is 17?

4. Is 21 is a strong probable prime to base 2?

(A) Yes

(B) No

5. Suppose that, when trying to codebreak some ciphertext that was encrypted using rectangular transposition, you find a "G box" that looks as follows, where ! indicates an entry that is much smaller than every other entry on the same row:

$$\begin{bmatrix} . & . & . & ! & . \\ . & . & ! & . & . \\ . & . & . & . & ! \\ . & . & . & . & . \\ ! & . & . & . & . \end{bmatrix}$$

Use the decrypting permutation suggested by this "G box" to decrypt the ciphertext EWART.

6. Alice and Bob agree to use affine encryption with $a = 3$ and $b = 8$. What is the decryption of the ciphertext letter B?

7. What is $\left(\dfrac{2}{7}\right)$?

(A) 1

(B) -1

(C) 0

8. On a distant alien planet, 10% of the population has a genetic condition which causes them to spontaneously combust at the age of 37. A test for this genetic condition given at birth has a 20% false positive rate and a 5% false negative rate.

Qwtqwt is a newborn alien on this planet who has just tested positive for this genetic condition. What is the probability that he will spontaneously combust at age 37?

9. Bob's Elgamal public key has $p = 31$, $g = 3$, and $h = 27$.

Alice wants to send Bob the message C. She generates an ephemeral key $y = 5$. What is the ciphertext that she sends Bob?

10. Using the matrix

$$A = \begin{bmatrix} 3 & 2 \\ 1 & 9 \end{bmatrix}$$

as a key, decrypt the ciphertext `CJDB`.

The numbers corresponding to the letters `CJDB` are 2, 9, 3, 1.

11. Eve has just intercepted a ciphertext with index of coincidence 1.3. Which of the following ciphers may have been used to encrypt this ciphertext?

(A) Rectangular transposition

(B) Affine cipher

(C) Playfair cipher

(D) Caesar cipher

12. Amélie rolls a six-sided dice. If it's 3 or less, she loses $1. If it's a 4, nothing happens. If it's 5 or more, she wins $2. Which of the following is most accurate?

(A) Amélie is expected to lose money

(B) Amélie is expected to break even

(C) Amélie is expected to win money

13. Find integers x and y such that
$\gcd(150, 90) = 150x + 90y$.