MA117 - Worksheet 3
Probability
Week 1, Thursday

Problem 1. You're playing a game with a friend which involves rolling a pair of dice each turn, and where rolling bigger numbers is better. On the first turn, you roll two 3s, and your friend rolls two 6s. On the second turn, you roll two 3s again, and your friend rolls two 6s again. You accuse your friend of cheating because rolling double 6s for two turns in a row is very unlikely. Your friend retorts that your rolls were just as unlikely. Is your friend right? Explain why or why not.

Problem 2. The 2010 American Community Survey estimates that 14.6% of Americans fall below the poverty line, 20.7% speak a language other than English at home, and 4.2% fall into both categories.

(a) What percentage of Americans fall below the poverty line and only speak English at home?
(b) Suppose a given American speaks a language other than English at home. What is the probability that they fall below the poverty line?
(c) Is the event that an American speaks a language other than English at home independent of the event that they fall below the poverty line?

Problem 3. There are 10 questions on a multiple-choice test, and each question has 4 options. You have not studied for the test at all, so you guess independently on each problem.

(a) What is the probability that you guess all of the answers correctly?
(b) What is the probability that you guess at least one correctly?
(c) What is the probability that the only question you get right is the very last one?

Problem 4. There are 10 questions on a multiple-choice test, and each question has 4 options. You studied very little for this test. For two of the questions, you're pretty confident you can rule out 2 of the possible answers; you then guess randomly between the remaining two options. For the remaining 8 questions, you just guess randomly out of all four options. What is your expected score on this exam?

Problem 5. Your friend hands you a coin and tells you to flip it. If you get heads, she will give you \$10. If you get tails, you will have to give her \$5. Seems like a great deal! But, unbeknownst to you, the coin your friend has handed you is actually biased and lands heads with probability 1/5.

(a) What is the expected value of your winnings?
(b) Calculate the standard deviation of your winninngs.

Problem 6. Ice cream usually comes in boxes with volume 48 fluid ounces, with a variance of 1. Ice cream scoops hold about 2 ounces, with a variance of 0.0625. What is the variance of the amount of ice cream left in the box after you scoop out one scoop of ice cream?

Problem 7. The distant planet of Ungli is home to a race of aliens who call themselves Ungliwallahs. Half of the population of Ungliwallahs have 20 fingers and the other half have 40 fingers.

Let X be the random variable modeling the number of fingers on a randomly chosen Ungliwallah. What is the expected value of X? What is the variance of X?

Problem 8. In the Kingdom of Okane, there are two types of coin in circulation called *kinka* and *ginka*. The masses of *kinka* have mean 2 g and standard deviation 0.1 g. The masses of *ginka* have mean 3 g and standard deviation 0.05 g. Okanemochi is the richest aristocrat in the Kingdom of Okane. Every morning, he fills his empty coin purse with exactly 10 *kinka* and 20 *ginka* before he heads out for his daily aristocratic errands. The coins he puts into his coin purse are chosen independently of one another.

(a) What is the expected total mass of the coins in Okanemochi's coin purse every morning?
(b) What is the standard deviation of the total mass of the coins in Okanemochi's coin purse every morning?

Problem 9. You roll two dice and take the absolute value of the difference of the two (eg, if you roll a 3 and 5, the result is $|3 - 5| = 2$). What is the *least* likely result? What is the *most* likely result? How much more likely is the most likely result than the least likely result?

Problem 10 (Challenging)**.** Suppose you have two biased coins: one lands heads with probability 1/10 and the other lands heads with probability 9/10. You put both of the coins into a bag, and then randomly pull one coin out. You flip that coin twice. Let A_1 be the event that the first flip is heads, and A_2 be the event that the second flip is heads. Are A_1 and A_2 independent?