Worksheet W2Tue: Complex Exponential

Problem 1. Show that $|e^z| \leqslant e^{|z|}$ for all $z \in \mathbb{C}$.

Problem 2. Find a formula for $\operatorname{Re} \exp(e^z)$ in terms of $x = \operatorname{Re}(z)$ and $y = \operatorname{Im}(z)$.

Problem 3. What can you say about e^{x+iy} for fixed y as $x \to \pm\infty$? What about for fixed x as $y \to \pm\infty$?

Problem 4. For what values of z is it that $\overline{e^{iz}} = e^{i\bar{z}}$?

Problem 5. For each of the following, draw a picture of the subset of $\mathbb{C}$ consisting of all $z \in \mathbb{C}$ satisfying the indicated condition, and then draw an image of the image of that subset under exp.

(a) $\operatorname{Re} z \in [-1, 1]$ and $\operatorname{Im} z \in [0, 2\pi]$.

(b) $\operatorname{Re} z \in [0, 1]$ and $\operatorname{Im} z \in [-\pi, 0]$.

(c) $\operatorname{Re} z = 1$.

(d) $\operatorname{Im} z = 1$.

(e) $\operatorname{Re} z = 2 \operatorname{Im} z$.

(f) $\operatorname{Re} z = (\operatorname{Im} z)^2$.

Problem 6. For each of the following, find a subset of $\mathbb{C}$ whose image under exp is the indicated set.

(a) $D[0, 1] \setminus \{0\}$

(b) $\mathbb{C} \setminus D[0, 1]$

(c) $D[0, 2] \setminus D[0, 1]$

Problem 7. For each of the following, find a formula involving exp for the function that the power series represents.

(a) $\displaystyle\sum_{k=0}^{\infty} \frac{z^{3k+1}}{k!}$

(b) $\displaystyle\sum_{k=1}^{\infty} \frac{2(2k-1)z^{2k-2}}{(k-1)!}$