

### Worksheet 3: Direct and Contrapositive Proofs, Division, GCD, Congruence

**Problem 1.** Prove that, if  $a$  is an integer such that  $5 \mid 2a$ , then  $5 \mid a$ .

**Problem 2.** Prove that  $5n^2 + 3n + 7$  is odd for every integer  $n$ .

**Problem 3.** Prove that every odd integer is the difference of two consecutive squares.

**Problem 4.** Show that the square of any integer cannot be congruent to 2 modulo 3.

**Problem 5.** For any integer  $n$ , show that either  $n$ ,  $n + 2$ , or  $n + 4$  must be divisible by 3.

**Problem 6.** Show that if  $n$  is an integer and  $n^2$  is not divisible by 4, then  $n$  must be odd.

**Problem 7.** Let  $a, b \in \mathbb{Z}$  are both nonzero. Show that  $\text{lcm}(a, b)$  divides any common multiple of  $a$  and  $b$ .

**Problem 8.** Suppose  $a$  and  $b$  are integers that are not both 0. Show that  $\text{gcd}(a, b) = \text{gcd}(a - b, b)$ .

**Problem 9.** For positive integers  $a$  and  $b$ , prove that  $\text{gcd}(a, b) \text{lcm}(a, b) = ab$ .

**Problem 10.** Let  $n$  be a positive integer. Show that, if  $a$  and  $b$  are integers such that  $a \equiv b \pmod{n}$ , then  $\text{gcd}(a, n) = \text{gcd}(b, n)$ .