Worksheet 2: Logic

Problem 1. Classify each of the following as "true statement," "false statement," "open sentence," or "none of the above."

(a) Every integer is odd.

(b) Every even integer is a real number.

(c) $\mathbb{N} \notin \mathscr{P}(\mathbb{N})$.

(d) The integer x is a multiple of 7.

(e) If $x, y, z \in \mathbb{R}$ and $xy = xz$, then $y = z$.

(f) If S is a finite set, then $\mathscr{P}(S)$ is finite.

(g) Does Sunny like cookies?

(h) My name is Sunny.

Problem 2. Express each of the following statements in the form $P \implies Q$, specifying what P and Q are. Then express the contrapositive and the converse of the statement in words. Finally, state (without proof) if the statement is true and also if its converse is true.

(a) For a function to be continuous, it is sufficient that it be differentiable.

(b) The series $\sum_{n=1}^{\infty} r^n$ converges if $|r| < 1$.

(c) An integer is divisible by 8 only if it is divisible by 4.

(d) For a real number a to be rational, it is necessary that $5a$ be rational.

Problem 3. Consider the statement "If every glorp is a slurp, then there exists a narp which isn't a yarp." What is its contrapositive? What is its converse? What is the contrapositive of the converse?

Problem 4. Write each of the following in English sentences. Then state whether they are true or false.

(a) $\forall x \in \mathbb{R}, \exists n \in \mathbb{N}, x^n \geqslant 0$

(b) $\exists n \in \mathbb{N}, \forall X \in \mathscr{P}(\mathbb{N}), |X| < n$

(c) $\forall n \in \mathbb{Z}, \exists m \in \mathbb{Z}, m = n + 5$

(d) $\exists m \in \mathbb{Z}, \forall n \in \mathbb{Z}, m = n + 5$

Problem 5. Negate the following statements.

(a) The square root of every prime number is irrational.

(b) If f is a polynomial of degree is greater than 2, then f' is not constant.

(c) There exists a real number a such that $a + x = x$ for every real number x.

(d) For every positive number ϵ, there exists a positive number δ such that $|x - a| < \delta$ implies $|f(x) - f(a)| < \epsilon$.

Problem 6. Construct a truth table for each of the following.

(a) $Q \vee R \iff (R \wedge Q)$

(b) $(P \wedge \neg P) \wedge Q$

Problem 7. Suppose the statement $(P \wedge Q) \vee R) \implies (R \vee S)$ is false. What are the truth values of P, Q, R, and S?

Problem 8. Does the truth value of $(P \vee \neg P) \implies (P \wedge \neg P)$ depend on the truth value of P?

Problem 9. Determine whether or not the two statements are logically equivalent.

$$\neg P \iff Q$$
$$(P \implies \neg Q) \wedge (\neg Q \implies P)$$

Problem 10. Determine whether or not the two statements are logically equivalent.

$$(P \implies Q) \vee R$$
$$\neg((P \wedge Q) \wedge \neg R)$$