Worksheet 10: Binary Exponentiation, Euler's Theorem

Problem 1. Calculate binary representations of the following numbers.

(a) 17

(b) 64

(c) 97

(d) 100

Problem 2. Formulate and prove a rule for determining if a number is divisible by 3 using the digits of the binary representation.

Problem 3. Calculate $\varphi(36000)$.

Problem 4. Find the units digit of 3^{100}.

Problem 5. Show that $17 \mid 11^{104} + 1$.

Problem 6. (a) Show that, if n is odd, then $\varphi(2n) = \varphi(n)$.

(b) Show that, if n is even, then $\varphi(2n) = 2\varphi(n)$.

Problem 7. Show that $\phi(n) = n/2$ if and only if $n = 2^e$ for some positive integer e.

Problem 8. Show that, if $\varphi(n) \mid n - 1$, then n is square-free (ie, all of the exponents in its prime factorization are 1).

Problem 9. Suppose $b_0, \ldots, b_r \in \{0, 1\}$ with $b_r = 1$ and let $k = b_0 + 2b_1 + 2^2 b_2 + \cdots + 2^r b_r$ be the number whose binary representation is $b_r \cdots b_0$. Write down a formula for the number of multiplications required when computing a^k for some a.

Problem 10. How many prime numbers are there such that p divides $2^{9^p} + 1$?