1. True or False?

Suppose $h : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear map, and there exists a basis B such that

$$\mathrm{Rep}_{B,B}(h) = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}.$$

Then there must exist a nonzero vector $v \in \mathbb{R}^2$ such that $h(v) = 2v$.

2. True or False?

Suppose $h : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear map, and there exist two bases B and C of $\mathbb{R}^2$ such that

$$\mathrm{Rep}_{B,C}(h) = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}.$$

Then there must exist a nonzero vector $v \in \mathbb{R}^2$ such that $h(v) = 2v$.

3. True or False?

If a square matrix A is similar to another square matrix B, then A^2 is also similar to B^2.

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Follow-up. What happens if we replace both instances of the word "similar" above with the word "equivalent"? In other words, is it true that if A and B are equivalent, then A^2 is equivalent to B^2?