1. Let V be the null space of the linear map $h : \mathcal{M}_{2\times 2} \to \mathbb{R}^2$ given by

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \mapsto \begin{pmatrix} c - 2b \\ a - 2b \end{pmatrix}$$

What is $\dim(V)$?

(A) 4

(B) 3

(C) 2

(D) 1

2. True or False?

Let $B = \langle 1, x, x^2 \rangle$ be the standard basis for $\mathcal{P}_2$ and C the standard basis for $\mathbb{R}^3$. Let $h : \mathcal{P}_2 \to \mathbb{R}^3$ be the linear map defined by

$$p \mapsto \begin{pmatrix} p(0) \\ p(1) \\ p(2) \end{pmatrix}.$$

Then

$$\mathrm{Rep}_{B,C}(h) = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \end{pmatrix}.$$

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Follow-up. Is h an isomorphism?

3. True or False?

There exists a surjective (ie, onto) linear map $h : \mathbb{R}^3 \to \mathbb{R}^2$ whose null space is 2 dimensional.