TEXTBOOK ERRATA

If you find a mistake in the textbook or in the solutions manual, please discuss it with me! I'll add it to this list (and attribute the discovery to you!). At the end of our class, we can send everything to the author.

Chapter Two.

- Exercise III.2.36 is a true statement, but the solution in the solutions manual is not quite correct: it is not true that, if we choose arbitrary bases for U and W, then one of the chosen basis elements of W must be in U. For example, consider the following vectors.

$$\vec{u}_1 = \begin{pmatrix}1\\0\\0\\0\\0\end{pmatrix} \quad \vec{u}_2 = \begin{pmatrix}0\\1\\0\\0\\0\end{pmatrix} \quad \vec{u}_1 = \begin{pmatrix}0\\0\\1\\0\\0\end{pmatrix}$$

$$\vec{w}_1 = \begin{pmatrix}0\\0\\0\\1\\0\end{pmatrix} \quad \vec{w}_2 = \begin{pmatrix}0\\0\\0\\0\\1\end{pmatrix} \quad \vec{w}_3 = \begin{pmatrix}1\\1\\1\\1\\1\end{pmatrix}$$

Note that the $\vec{u}$'s form a linearly independent set, as do the $\vec{w}$'s. This means that the $\vec{u}$'s form a basis B_U for $U = [\vec{u}_1, \vec{u}_2, \vec{u}_3]$, and the $\vec{w}$'s form a basis B_W for $W = [\vec{w}_1, \vec{w}_2, \vec{w}_3]$. But none of the $\vec{w}$'s is in U.

One possible correct solution would be something like the following. Suppose that $\langle \vec{u}_1, \vec{u}_2, \vec{u}_3 \rangle$ and $\langle \vec{w}_1, \vec{w}_2, \vec{w}_3 \rangle$ are bases for U and W, respectively. Then

$$\langle \vec{u}_1, \vec{u}_2, \vec{u}_3, \vec{w}_1, \vec{w}_2, \vec{w}_3 \rangle$$

is a list of 6 elements in $\mathbb{R}^5$, which is 5-dimensional, so this list must be linearly dependent by Corollary III.2.11. In other words, there exist $a_1, a_2, a_3, b_1, b_2, b_3$ not all zero such that

$$a_1\vec{u}_1 + a_2\vec{u}_2 + a_3\vec{u}_3 + b_1\vec{w}_1 + b_2\vec{w}_2 + b_3\vec{w}_3 = 0.$$

If $a_1 = a_2 = a_3 = 0$, then this equation would yield a linear dependence relation among $\vec{w}_1, \vec{w}_2, \vec{w}_3$, which is a contradiction. Thus not all of the a's are 0. Define

$$\vec{u} = a_1\vec{u}_1 + a_2\vec{u}_2 + a_3\vec{u}_3.$$

Since $\vec{u}_1, \vec{u}_2, \vec{u}_3$ are linearly independent and not all of the a's are 0, we know that $\vec{u} \neq 0$.

Clearly $\vec{u} \in [\vec{u}_1, \vec{u}_2, \vec{u}_3] = U$. On the other hand, the earlier linear dependence relation shows that

$$\vec{u} = -b_1\vec{w}_1 - b_2\vec{w}_2 - b_3\vec{w}_3 \in [\vec{w}_1, \vec{w}_2, \vec{w}_3] = W$$

so $\vec{u}$ is a nonzero vector in $U \cap W$, proving that $U \cap W$ is nontrivial.

Chapter Three.

- In exercise III.1.27, one should assume that h is both one-to-one *and onto* (because otherwise $h(B)$ will just be a basis for $\mathscr{R}(h)$, not all of W).
- In the last line of example III.2.1, the symbol $\vec{w}$ appears but is never defined. It should probably read $h(\vec{v})$ instead.
- In example V.2.1, towards the bottom of page 260, it should say "Find**ing** that matrix is easy, and we have a formula..."
- (Thanks to Natalie Sarver and Brooke Veale) The solution to exercise IV.24(b) should be
$$\begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix}.$$
Since the problem is asking for the result of multiplying a permutation matrix on the right, the answer should just involve an interchange of columns.
- (Thanks to Michael Bennett) In the solution to exercise V.2.15, the arrow diagram given in (a) should translate to the matrix equation $\hat{H} = PHQ$ where $Q = \mathrm{Rep}_{\hat{B},B}(\mathrm{id})$ and $P = \mathrm{Rep}_{D,\hat{D}}(\mathrm{id})$. (In the solution to (b), the definitions of P and Q are flipped.)