Day 28

Review

1. How many homomorphisms $D_5 \to Z_3$ are there?

2. Is it true that $\mathbf{R}^* = \mathbf{R}^+ \times \{1, -1\}$?

3. Let $\phi : \mathbf{R} \oplus \mathbf{R} \to \mathbf{R}^*$ be the function $\phi(x, y) = e^{x+y}$. Show that ϕ is a homomorphism. What is $\ker \phi$?

4. Show that the factor group $\mathbf{R}/\mathbf{Z}$ has an element of order n for every positive integer n.

5. Let $G = Z_6 \oplus Z_3$ and let $H = \langle(2, 0)\rangle$. Is the factor group G/H cyclic? If so, identify a generator.

6. Is $\langle (1\ 2\ 3) \rangle$ a normal subgroup of S_4?

7. Is $H = \{(a, b) \mid a, b \in \mathbf{R}, 2a + 3b = 0\}$ a subgroup of $\mathbf{R} \oplus \mathbf{R}$?

9. Is it true that $U(n)$ has an element of order 2 for all $n > 2$?

10. For how many integers $n \geq 2$ is it the case that $(1\ 2) \in Z(S_n)$?

11. Let H be the subset of $G = \mathrm{GL}(2, \mathbf{R})$ consisting of all matrices that are either of the form

$$\begin{bmatrix} a & 0 \\ 0 & a^{-1} \end{bmatrix}$$

for some $a \in \mathbf{R}^*$ or of the form

$$\begin{bmatrix} 0 & -b \\ b^{-1} & 0 \end{bmatrix}$$

for some $b \in \mathbf{R}^*$. Is H is a subgroup of G?

12. Is there a group G which contains a proper subgroup H such that $H \approx G$?