MA117 - Worksheet 3
Probability
Week 1, Thursday

Problem 1. You're playing a game with a friend which involves rolling a pair of dice each turn, and where rolling bigger numbers is better. On the first turn, you roll two 3s, and your friend rolls two 6s. On the second turn, you roll two 3s again, and your friend rolls two 6s again. You accuse your friend of cheating because rolling double 6s for two turns in a row is very unlikely. Your friend retorts that your rolls were just as unlikely. Is your friend right? Explain why or why not.

Problem 2. You roll two dice and take the absolute value of the difference of the two (eg, if you roll a 3 and 5, the result is $|3 - 5| = 2$). What is the *least* likely result? What is the *most* likely result? How much more likely is the most likely result than the least likely result?

Problem 3. The 2010 American Community Survey estimates that 14.6% of Americans fall below the poverty line, 20.7% speak a language other than English at home, and 4.2% fall into both categories.

(a) What percentage of Americans fall below the poverty line and only speak English at home?
(b) Suppose a given American speaks a language other than English at home. What is the probability that they fall below the poverty line?
(c) Is the event that an American speaks a language other than English at home independent of the event that they fall below the poverty line?

Problem 4. Lupus is a medical phenomenon where antibodies that are supposed to attack foreign cells to prevent infections instead see plasma proteins as foreign bodies, leading to a high risk of blood clotting. It is believed that 2% of the population suffer from this disease. The test is 98% accurate if a person actually has the disease. The test is 74% accurate if a person does not have the disease. There is a line from the Fox television show House that is often used after a patient tests positive for lupus: "It's never lupus." Do you think there is truth to this statement? Use appropriate probabilities to support your answer.

Problem 5. There are 10 questions on a multiple-choice test, and each question has 4 options. You have not studied for the test at all, so you guess independently on each problem.

(a) What is the probability that you guess all of the answers correctly?
(b) What is the probability that you guess at least one correctly?
(c) What is the probability that the only question you get right is the very last one?

Problem 6 (Challenging)**.** Suppose you have two biased coins: one lands heads with probability 1/10 and the other lands heads with probability 9/10. You put both of the coins into a bag, and then randomly pull one coin out. You flip that coin twice. Let A_1 be the event that the first flip is heads, and A_2 be the event that the second flip is heads. Are A_1 and A_2 independent?