Problem. Suppose the mass of a gold coin is normally distributed with mean 30 g and standard deviation 1 g. You pick out a stack of 100 gold coins randomly and independently of each other. What is the standard deviation of the total mass of all 100 gold coins? Explain briefly.

Solution. Let X_i denote the mass of the ith gold coin. Then $X = X_1 + \cdots + X_{100}$ models the mass of all 100 gold coins. Since $X_1, \ldots, X_{100}$ are all independent, we have

$$\text{Var}(X) = \text{Var}(X_1) + \cdots + \text{Var}(X_{100}) = 1^2 + \cdots + 1^2 = 100$$

using the formula from p.~123 of the textbook. So the standard deviation is $\sqrt{100} = 10$ g.

Problem. In the Kingdom of Okane, there are two types of coin in circulation called *kinka* and *ginka*. The masses of *kinka* are normally distributed with mean 2 g and standard deviation 0.1 g. The masses of *ginka* is normally distributed with mean 3 g and standard deviation 0.05 g. Okanemochi is the richest aristocrat in the Kingdom of Okane. Every morning, he fills his empty coin purse with exactly 10 *kinka* and 20 *ginka* before he heads out for his daily aristocratic errands. The coins he puts into his coin purse are chosen independently of one another. What is the standard deviation of the total mass of the coins in Okanemochi's coin purse every morning?

Solution. Let $K_1, \ldots, K_{10}$ be random variables representing the masses of the 10 *kinka* coins that Okanemochi puts in his coin purse; we know that $\text{Var}(K_i) = 0.01$ for all i. Let $G_1, \ldots, G_{20}$ random variables representing the masses of the 20 *ginka* coins Okanemochi puts in his coin purse; we know that $\text{Var}(G_i) = 0.0025$ for all i. The total mass P of the coin purse is

$$P = K_1 + \cdots + K_{10} + G_1 + \cdots + G_{20}.$$

Since the masses of the coins are all independent, we have

$$\begin{aligned}
\text{Var}(P) &= \text{Var}(K_1) + \cdots + \text{Var}(K_{10}) + \text{Var}(G_1) + \cdots + \text{Var}(G_{20}) \\
&= 10 \cdot 0.01 + 20 \cdot 0.0025 \\
&= 0.1 + 0.05 \\
&= 0.15
\end{aligned}$$

Thus the standard deviation is $\sqrt{0.15} \approx 0.39$ g.