> Can using the RREF method for reducing a Gröbner basis be used in all cases?

No: row reduction is a very special case! Buchberger's algorithm is a very general process for finding a (reduced) Gröbner basis for the ideal generated by an arbitrary list of polynomials. In the very special situation where the polynoials you start with are all *linear*, Buchberger's algorithm is basically the same thing as row reduction.

In other words, I think it's useful to think about Buchberger's algorithm as a (very significant!) generalization of row reduction.

> When considering the possible pairs to apply the S-polynomial to in an ideal, say $I = \langle f_1, f_2 \rangle$, is $S(f_1, f_2) = S(f_2, f_1)$? If not, are we only considering i, j where $i < j$ for $S(f_i, f_j)$ always? Or should we always consider $i > j$ as well?

For the first part, see the reading question responses from yesterday: it is not true that $S(f_1, f_2) = S(f_2, f_1)$, but it is true that $S(f_1, f_2) = -S(f_2, f_1)$.

For the second part, it is sufficient to consider $i < j$. When $i > j$, $S(f_i, f_j) = -S(f_j, f_i)$, so if you've already guaranteed that the remainder of $S(f_j, f_i)$ will be 0 when you divide by $(f_1, \ldots, f_t)$, the same will be true of $S(f_i, f_j)$.

> Is there another way to check if a Gröbner basis is reduced except by the definition?

I don't know of another way, but the definition of a reduced Gröbner basis is not very difficult to check in practice I think. You just need to go through your list $g_1, \ldots, g_t$, and for each g_i, figure out if any of the terms of g_i is divisible by $LT(g_j)$ for some $j \neq i$. When t is not too big, this is even not so hard to do by hand, and computers can certainly do this very very quickly.

You might also note that the *proof* of theorem 5 actually describes an algorithm for constructing the reduced Gröbner basis out of a minimal Gröbner basis.