

I would like to see a concrete example of an ideal and its generating set.

Consider the polynomial ring $k[x]$ in just one variable x . What is the ideal $\langle x \rangle$ concretely? I claim that

$$\langle x \rangle = \left\{ \sum a_i x^i \in k[x] \mid a_0 = 0 \right\}.$$

In other words, I claim that $\langle x \rangle$ is precisely the set of all polynomials which have no constant term.

To prove this, suppose first that we have a polynomial $f = \sum a_i x^i$ where $a_0 = 0$. That means that

$$f = a_1 x + a_2 x^2 + \cdots + a_n x^n = \underbrace{(a_1 + a_2 x + \cdots + a_n x^{n-1})}_h x = hx$$

so that means that $f \in \langle x \rangle$ (definition 1.4.2 says that $\langle x \rangle$ is precisely all multiples of x , and we've just shown that any f that has no constant term is in fact a multiple of x).

Conversely, suppose $f \in \langle x \rangle$. That means that $f = hx$ for some polynomial $h \in k[x]$. Let us write $h = a_0 + a_1 x + \cdots + a_n x^n$. Then

$$f = hx = a_0 x + a_1 x^2 + \cdots + a_n x^{n+1}$$

so f has no constant term.

How should we plot the ideal for affine varieties?

A central observation in algebraic geometry is that you can go back and forth between algebra and geometry. Varieties are on the geometry side of this dictionary. Ideals are on the algebra side.

We can plot varieties since varieties are geometric objects. It doesn't really make sense to "plot ideals" since ideals are algebraic objects (that being said, we can draw pictures of *some* ideals, like the monomial ideals we'll discuss in section 2.4).

Is an ideal a normal subgroup?

Yes. In an abelian group, every subgroup is normal. A ring is an abelian group under addition, and an ideal is a subgroup.