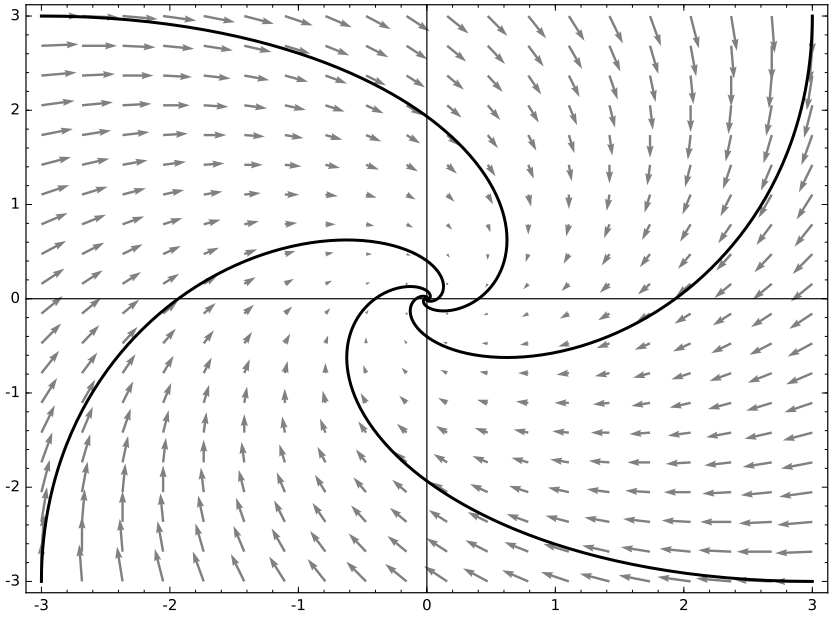
1. A is a matrix of real numbers and the phase portrait of a system $\vec{x}' = A\vec{x}$ is depicted to the right. Which of the following is a true statement about A?



(A) It has complex eigenvalues with positive real part.

(B) It has complex eigenvalues with negative real part.

(C) It has a deficient negative eigenvalue.

(D) None of the above.

2. True or False?

Suppose the matrix

$$\begin{bmatrix} 0 & 1 \\ b & a \end{bmatrix}$$

has a repeated eigenvalue λ. Then $\lambda = a/2$.

3. Suppose a 1 kg mass is attached to a wall by a spring of stiffness k and slides around on a surface with friction 2 kg/s, so that the equation of motion is governed by the ODE

$$x'' + 2x' + kx = 0.$$

For which of the following values of k is this system overdamped?

(a) $k = 0.5$ kg/s^2

(b) $k = 1$ kg/s^2

(c) $k = 2$ kg/s^2

(d) None of the above

4. Which of the following is a basis for the solution space of the second order homogeneous linear ODE $x'' - x' - 6x = 0$?

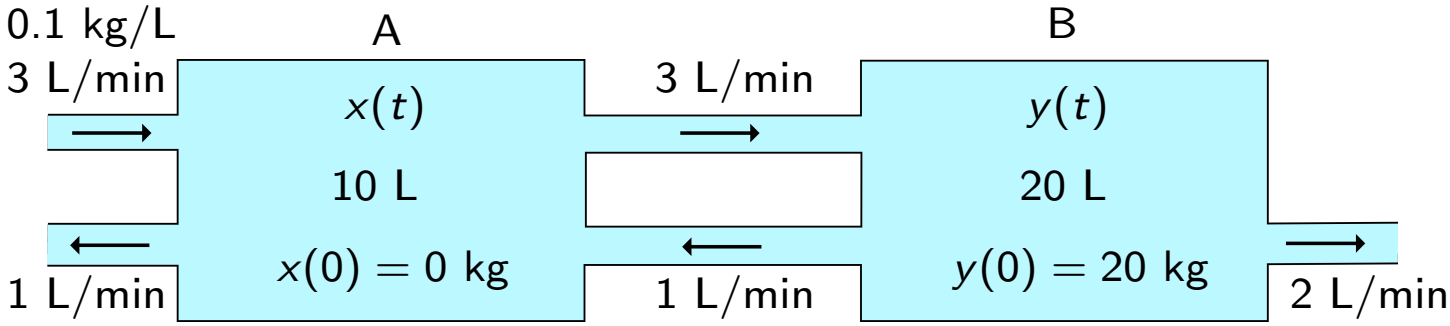
(A) e^{3t}, e^{-2t}

(B) $e^{3t}, -2e^{3t}$

(C) $e^{3t} - e^{-2t}, e^{3t} + e^{-2t}$

(D) More than one of the above

Salt water tanks! The input to tank A has a salt concentration of 0.1 kg/L.



5. If you were to solve the linear nonhomogeneous system

$$\begin{bmatrix} x \\ y \end{bmatrix}' = A \begin{bmatrix} x \\ y \end{bmatrix} + \vec{f}$$

using the method of undetermined coefficients, what form would you guess for the particular solution?

(A) $\vec{a}$

(B) $\vec{a}t + \vec{b}$

(C) Neither of the above

Consider the following matrix.

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

i is a deficient eigenvalue of A (with deficiency 1), $v_1 = (-i, 1, 0, 0)$ is an eigenvector, and $v_2 = (1, 0, 2, 2i)$ is a generalized eigenvector such that $(A - iI)v_2 = v_1$.

6. What is the general solution to the system $\vec{x} = A\vec{x}$?

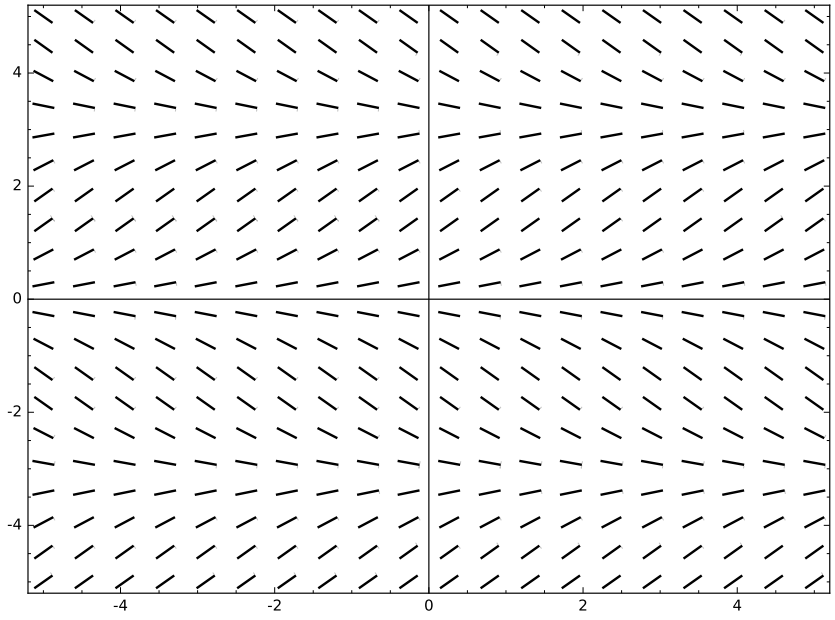
7. Which of the following is a true statement about the critical points of the autonomous ODE $y' = y^2 - 2y + 1$?

(A) It has more than one critical point.

(B) It has exactly one critical point, and it is unstable.

(C) It has exactly one critical point, and it is stable.

8. Which of the following first order ODEs could the depicted slope field belong to?



(A) $y' = \sin(y)$

(B) $y' = y^2$

(C) $y' = x + y$

(D) None of the above

9. The first order ODE $x' = \sin(t)x$ is...

(A) Separable and linear.

(B) Separable but not linear.

(C) Linear but not separable.

(D) Neither separable not linear.

10. True or False?

The initial value problem

$$y' = xy^2 - y, \quad y(0) = 2$$

has a unique solution.

11. Suppose y is a function such that $y' = y^2 - 1$ and $y(0) < 1$. What is $\lim_{x \to \infty} y(x)$?

(A) -1

(B) 0

(C) 1

(D) None of the above

12. True or False?

The initial value problem

$$(y - x^2)y' = 0, \quad y(0) = 0$$

has a unique solution.

13. Suppse you're solving the nonhomogeneous second order ODE

$$x'' + 4x = \cos(2t)$$

using the method of undetermined coefficients. What form should you guess for the particular solution?

(A) $x_p = a\cos(2t)$

(B) $x_p = a\cos(2t) + b\sin(2t)$

(C) $x_p = at\cos(2t) + bt\sin(2t)$

(D) None of the above

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(C) $x_p = at\cos(2t) + bt\sin(2t)$

(D) None of the above

Remark. Interpret using resonant frequencies.

14. How might you go about solving the first order ODE $y' + 3y = e^x$?

(A) Separating variables.

(B) Integrating factors.

(C) Neither of the above.

15. If you're solving the ODE $y' + 3y = e^x$ using integrating factors, what is the integrating factor?

(A) e^{3x}

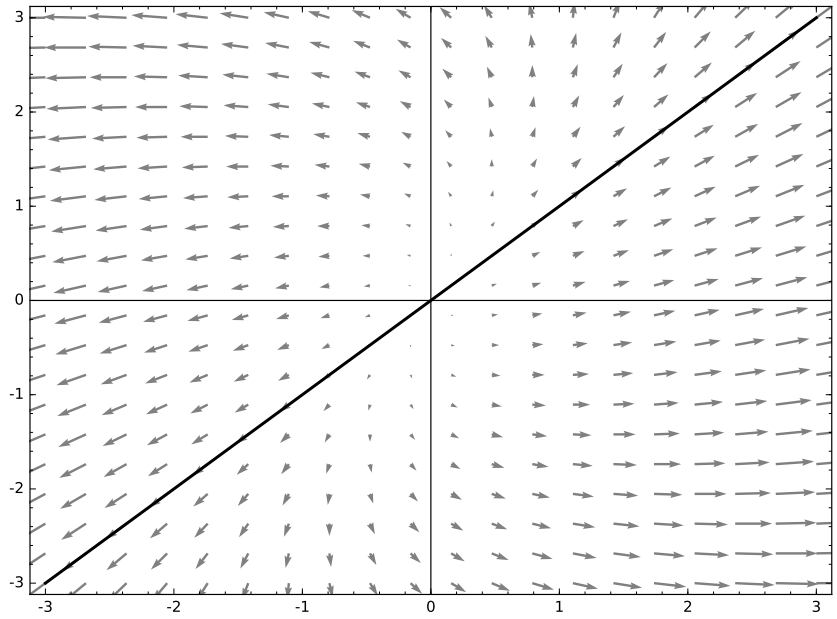
(B) e^{-3x}

(C) e^3

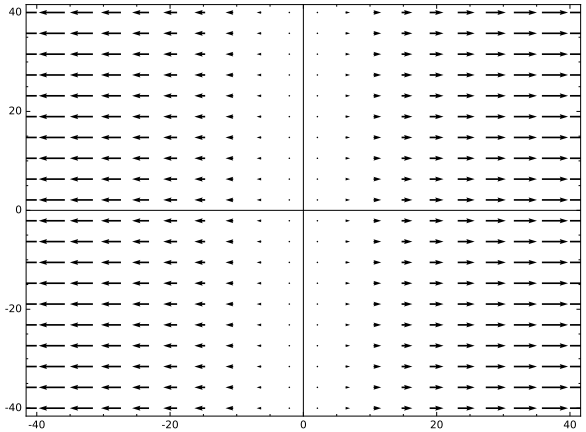
(D) None of the above

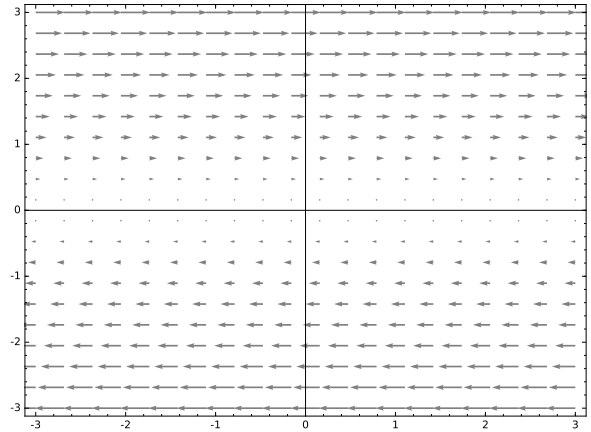
16. True or False?

If the depicted phase portrait belongs to a system $\vec{x}' = A\vec{x}$ for a 2×2 matrix A, then A must have a positive deficient eigenvalue.



17. Consider the following two phase portraits, which belong to the linear systems $\vec{x}' = A\vec{x}$ and $\vec{x}' = B\vec{x}$, respectively.





Which of the following is true?

(A) A has $\lambda = 0$ as a complete eigenvalue.

(B) B has $\lambda = 0$ as a deficient eigenvalue.

(C) Both the above.

(D) None of the above.

18. True or False?

If X is any fundamental matrix solution for a linear homogeneous system $\vec{x}' = A\vec{x}$, then

$$X' = AX.$$

19. True or False?

If X is any fundamental matrix solution for a linear homogeneous system $\vec{x}' = A\vec{x}$, then

$$X' = XA.$$

Follow-up. What about $X = e^{At}$?

20. Suppose a 1 kg mass is attached to a wall by a spring of stiffness k and slides around on a surface with friction 2 kg/s, so that the equation of motion is governed by the ODE

$$x'' + 2x' + kx = 0.$$

Suppose further that the system is underdamped and that it oscillates with natural frequency 2 rad/s. What must k be?

(a) $k = 2$ kg/s^2

(b) $k = 3$ kg/s^2

(c) $k = 4$ kg/s^2

(d) None of the above