Let $A = \begin{bmatrix} 5 & 4 \\ 1 & 8 \end{bmatrix}$. The eigenvalues of A are $\lambda_1 = 4$ and $\lambda_2 = 9$, with corresponding eigenvectors $\vec{v}_1 = \begin{bmatrix} -4 \\ 1 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

1. True or False?

$$e^{tA} = \begin{bmatrix} -4 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} e^{4t} & 0 \\ 0 & e^{9t} \end{bmatrix} \begin{bmatrix} -4 & 1 \\ 1 & 1 \end{bmatrix}^{-1}$$

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2. True or False?

If P is any invertible 2×2 matrix of constants, then

$$X(t) = \begin{bmatrix} -4e^{4t} & e^{9t} \\ e^{4t} & e^{9t} \end{bmatrix} P$$

is a fundamental solution matrix for the system $\vec{x}\,' = A\vec{x}$.

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Follow-up. What happens when P is *not* invertible?

3. True or False?

The matrix $\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$ is diagonalizable.

4. True or False?

If $N = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$, then

$$e^N = I + N + \frac{N^2}{2}.$$

5. True or False?

If $P = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$, then $e^P = \begin{bmatrix} e^2 & e^2 \\ 0 & e^2 \end{bmatrix}$.

Hint. Recall that $e^{A+B} = e^A e^B$ if $AB = BA$, and notice that

$$P = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}.$$

6. Compute e^A for the following matrices.

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

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