1. Consider the subspace

$$U = \left\{ \begin{bmatrix} a \\ a+b \\ b \\ a+b \end{bmatrix} : a, b \in \mathbb{R} \right\}$$

inside $\mathbb{R}^4$. What is $\dim(U)$?

(a) 1

(b) 2

(c) 3

(d) None of the above

Let $C^\infty(\mathbb{R})$ be the vector space whose elements are infinitely differentiable real-valued functions with domain all real numbers.

2. True or False?

Consider the three functions

$$f(x) = e^{2x}, g(x) = e^{3x}, \text{ and } h(x) = e^{4x}.$$

Then f, g, h is a linearly independent list in $C^\infty(\mathbb{R})$.

Let $C^\infty(\mathbb{R})$ be the vector space whose elements are infinitely differentiable real-valued functions with domain all real numbers.

3. True or False?

If $f(x) = \sin(2x)$ and $g(x) = \cos(2x)$, then f, g is linearly independent in $C^\infty(\mathbb{R})$.

4. Consider the matrix $A = \begin{pmatrix} -2 & -1 \\ 5 & 2 \end{pmatrix}$. Which of the following is true?

(A) A has just one eigenvalue of algebraic multiplicity 2.

(B) A has two distinct eigenvalues which are both real.

(C) A has two distinct eigenvalues which are complex conjugates.

(D) None of the above.

5. True or False?

The following matrix is diagonalizable.

$$\begin{pmatrix} 1 & 3 \\ 1 & 3 \end{pmatrix}$$

6. True or False?

If A is a square matrix such that $A^3 = 0$, then A must have exactly one eigenvalue.