1. If we rewrite the linear system of ODEs

$$\begin{cases} x'' = t^2 x + y' + x' \\ y''' = tx' + \sin(t)y \end{cases}$$

as a first order system $\vec{x}\,' = A\vec{x}$ for a matrix A, what are the dimensions of the matrix A?

(A) 2×2

(B) 3×3

(C) 4×4

(D) 5×5

2. True or False?

The phase portrait of the system $\vec{x}' = \begin{bmatrix} 1 & a \\ a & 1 \end{bmatrix} \vec{x}$ can never be a sink.

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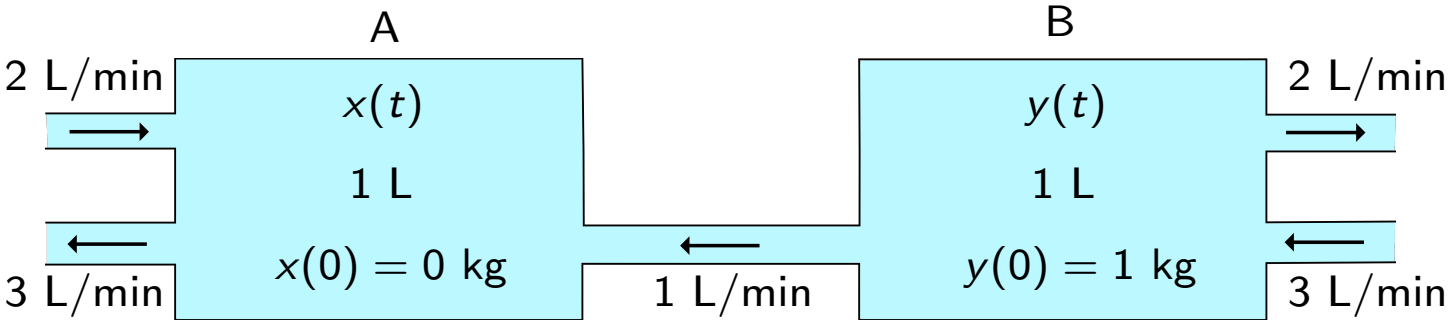
Follow-up. What does the phase portrait look like for various possiblities of a?

More salt-water tanks! Hooray! In the diagram below, the inputs to both tanks A and B are pure water.



3. If A is the matrix such that $\dfrac{d}{dt}\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = A\begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$, which of the following is true?

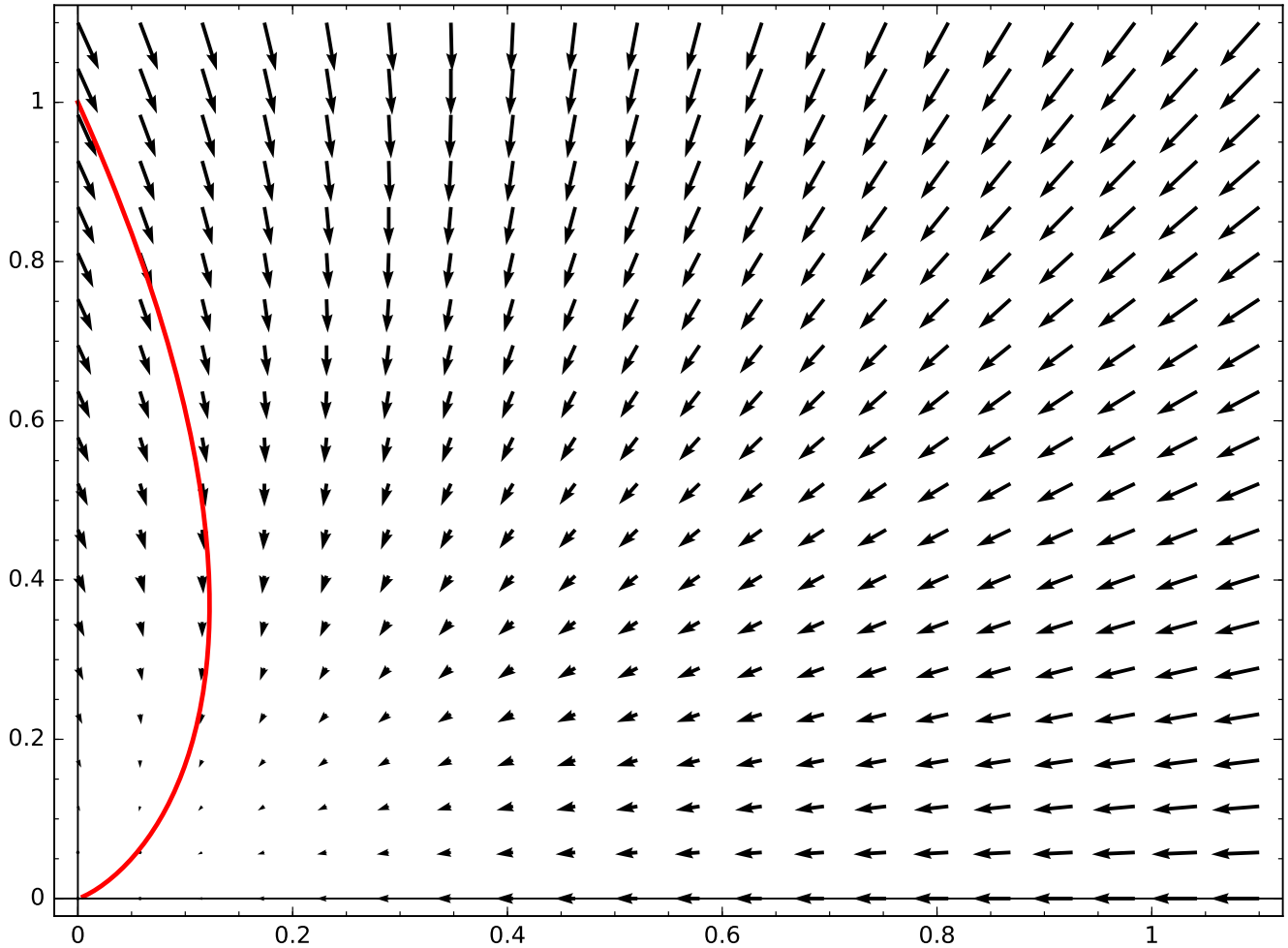
(A) A has a repeated deficient eigenvalue.

(B) A has a repeated complete eigenvalue.

(C) A has two distinct real eigenvalues.

(D) A has two distinct complex eigenvalues.

Here's a phase portrait for the previous problem.



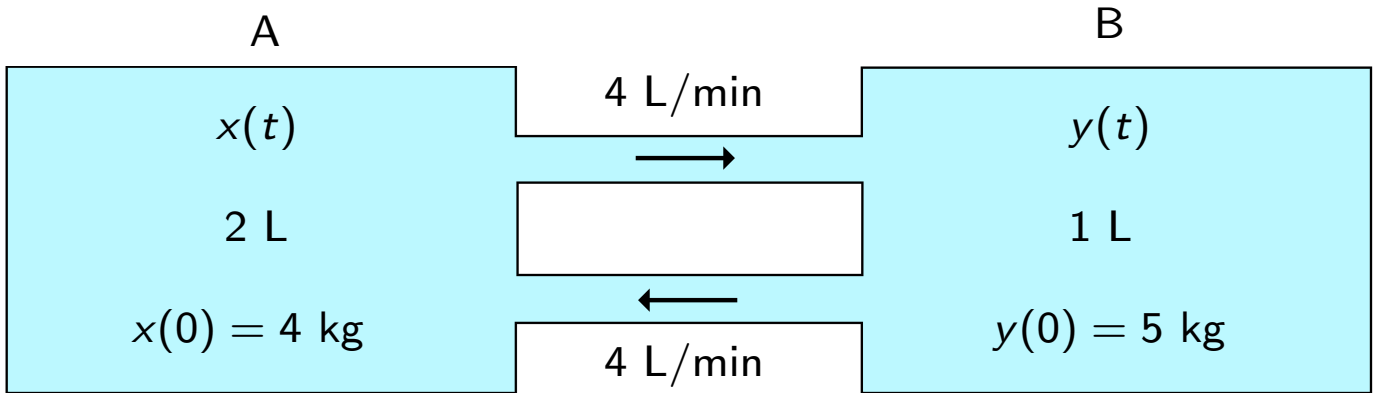
If an object of mass $m = 1$ kg is attached to a wall by a spring of stiffness $k = 20$ kg/s^2 and slides around on a surface with coefficient of friction $b = 4$ kg/s, the displacement x of the object from its equilibrium position is described by the ODE

$$x'' + 4x' + 20x = 0.$$

4. True or False?

$$\lim_{t \to \infty} x(t) = 0.$$

More interconnected salt-water tanks!



5. True or False?

$$\lim_{t\to\infty} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$$

6. Consider the matrix $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$.

Which of the following vectors is a generalized eigenvector of A but *not* an eigenvector?

(A) $\vec{e}_1 = (1, 0, 0)$

(B) $\vec{e}_2 = (0, 1, 0)$

(C) $\vec{e}_3 = (0, 0, 1)$

(D) None of the above