1. Does there exist a *smooth* function $f : \mathbb{R} \to \mathbb{R}$ such that $\mathrm{supp}(f) = \mathbb{R}$ and $f(1/n) = 0$ for all positive integers n? If so, provide an explicit example with justification. If not, prove that there cannot be such a function.

Definition. If I is an open interval, a function $f : I \to \mathbb{R}$ is said to be *concave up* if

$$\frac{f(b) - f(a)}{b - a} \leq \frac{f(c) - f(b)}{c - b}$$

for all $a < b < c$ in I. (Can you draw a picture of this definition?)

2. Suppose $f : I \to \mathbb{R}$ is differentiable. Prove that f is concave up if and only if f' is increasing.

Remark. Suppose f is twice differentiable. Then the above shows that f is concave up iff f' is increasing iff $f'' \geq 0$, which is a fact you probably remember learning in calculus! But the above fact is more general, since it only requires that f be differentiable (rather than twice differentiable).