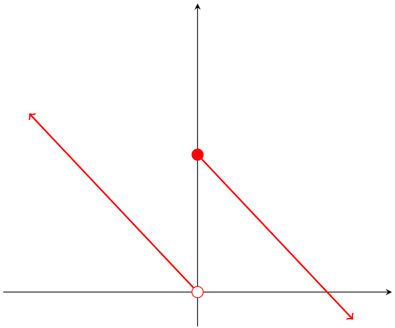
1. True or False?

Let $f : \mathbb{R} \to \mathbb{R}$ be the function

$$f(x) = \begin{cases} 1 - x & \text{if } x \geq 0 \\ -x & \text{if } x < 0, \end{cases}$$

whose graph is depicted to the right. Then f has the intermediate value property.



2. True or False?

Suppose $f : \mathbb{R} \to \mathbb{R}$ is differentiable and $\lim_{x\to\infty} f'(x) = 0$. If $g(x) = f(x+1) - f(x)$, then

$$\lim_{x\to\infty} g(x) = 0.$$

3. True or False?

Suppose $f : \mathbb{R} \to \mathbb{R}$ is differentiable and there exists $M > 0$ such that $|f'(x)| \leq M$ for all $x \in \mathbb{R}$. Then there exists $\epsilon > 0$ such that the function $g : \mathbb{R} \to \mathbb{R}$ given by

$$g(x) = x + \epsilon f(x)$$

is strictly increasing.

4. True or False?

Suppose a sequence of differentiable functions $f_n : \mathbb{R} \to \mathbb{R}$ converges uniformly and $f = \lim f_n$ is differentiable. Then $f' = \lim f_n'$.

5. True or False?

Suppose I is an open interval and $f : I \to \mathbb{R}$ is differentiable. If $a \in I$ and $\lim_{x \to a} f'(x)$ exists, then this limit must equal $f'(a)$.