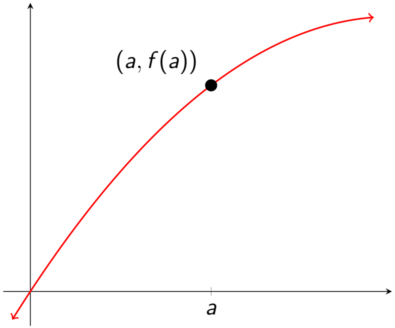
1. True or False?

Suppose f is the function whose graph is depicted in red on the right. Then there exists $\epsilon > 0$ such that

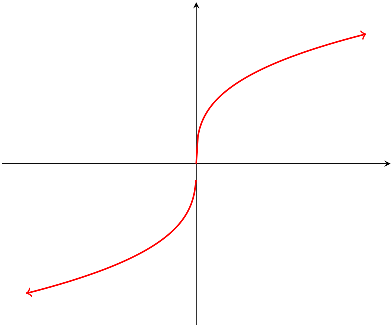
$$f'(a) < \frac{f(a+h) - f(a)}{h}$$

for all h such that $0 < h < \epsilon$.



2. True or False?

The function f whose graph is depicted below is differentiable at 0.



3. True or False?

$\sin(x) = o(|x|)$ as $x \to 0$.

Hint. You might use l'Hôpital's rule (all you need is the "weak version" you proved for the reading assignment yesterday), and standard rules for differentiation you learned in calculus (which are in your reading for tonight and tomorrow night).

4. True or False?

$\sin(x) - x = o(|x|)$ as $x \to 0$.

5. True or False?

Suppose $f : \mathbb{R} \to \mathbb{R}$ is a function and $a \in \mathbb{R}$. If f is differentiable at a, then

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a-h)}{2h}.$$

6. True or False?

Suppose $f : \mathbb{R} \to \mathbb{R}$ is a function and $a \in \mathbb{R}$. If

$$\lim_{h\to 0} \frac{f(a+h) - f(a-h)}{2h}$$

exists, then f is differentiable at a.

6. True or False?

Suppose $f : \mathbb{R} \to \mathbb{R}$ is a function and $a \in \mathbb{R}$. If

$$\lim_{h \to 0} \frac{f(a+h) - f(a-h)}{2h}$$

exists, then f is differentiable at a.

Follow-up. What if f is assumed to be *continuous* at a?

7. True or False?

Suppose $f : \mathbb{R} \to \mathbb{R}$ is a function such that

$$|f(a+h) - f(a)| \leq h^2$$

for all $a, h \in \mathbb{R}$. Then f is differentiable and $f' = 0$.

7. True or False?

Suppose $f : \mathbb{R} \to \mathbb{R}$ is a function such that

$$|f(a+h) - f(a)| \leq h^2$$

for all $a, h \in \mathbb{R}$. Then f is differentiable and $f' = 0$.

Follow-up. What happens if h^2 is replaced by h? By h^3?

The *euclidean norm* of a vector $(h, k) \in \mathbb{R}^2$, denoted $|(h, k)|_2$, is defined by

$$|(h, k)|_2 = \sqrt{h^2 + k^2}.$$

8. True or False?

$h^2 + k^2 = o(|(h, k)|_2)$ as $(h, k) \to 0$ in $\mathbb{R}^2$.

The *max norm* of a vector $(h, k) \in \mathbb{R}^2$, denoted $|(h, k)|_\infty$, is defined by

$$|(h, k)|_\infty = \max\{|h|, |k|\}.$$

9. True or False?

$h^2 + k^2 = o(|(h, k)|_\infty)$ as $(h, k) \to 0$ in $\mathbb{R}^2$.