1. True or False?

There exists an $\epsilon > 0$ such that, for every $(a, b) \in B(0, \epsilon)$, the system of equations

$$\begin{cases} x - y^2 = a \\ x^2 + y + 3y^3 = b \end{cases}$$

has a unique solution $(x, y) \in B(0, \epsilon)$.

2. True or False?

Suppose $f : \mathbb{R}^2 \to \mathbb{R}$ is a function such that $\partial_1 f, \partial_2 f$ both exist. If $g : \mathbb{R} \to \mathbb{R}^2$ is differentiable, then $f \circ g$ is also differentiable.

3. Suppose $f : \mathbb{R}^2 \to \mathbb{R}^2$ is given by

$$f(x, y) = (ye^x, xe^y).$$

Observe that $f(0, 1) = (1, 0)$. Show that there exists an open neighborhood U of $(0, 1)$ such that $f|_U$ is injective and $f(U)$ is open, and then calculate $((f|_U)^{-1})'(1, 0)$.