1. True or False?

There exists a non-constant differentiable function $f : \mathbb{R}^2 \to \mathbb{R}$ such that, for some fixed nonzero $h \in \mathbb{R}^2$, we have

$$df_a(h) = 0$$

for all points $a \in \mathbb{R}^2$.

2. True or False?

The function $f : \mathbb{R}^2 \to \mathbb{R}$ given by $f(x, y) = |xy|$ is differentiable at the origin.

3. Draw pictures of the function $f(x, y) = (x, y^2)$ until you can see why this function is called a “fold.”

Then calculate the set of all critical points of f.

4. True or False?

If $f : \mathbb{R}^3 \to \mathbb{R}$ is differentiable at 0 and

$$f'(0) = \begin{bmatrix} 3 & -1 & -2 \end{bmatrix},$$

then $\partial_{(1,1,1)} f(0) = 0$.

5. True or False?

If $f : \mathbb{R}^2 \to \mathbb{R}$ is given by

$$f(x, y) = 2(x-1) - 3y + y^2,$$

then $df_{(1,0)}(h, k) = 2h - 3k$.

Single variable review!

Recall. If I is an open interval, a function $f : I \to \mathbb{R}$ is defined to be *concave up* if

$$\frac{f(y) - f(x)}{y - x} \leq \frac{f(z) - f(y)}{z - y}$$

for all $x < y < z$ in I.

6. Show that, if $f : I \to \mathbb{R}$ is concave up and $f'(a) = 0$, then a is an absolute minimum of f.

Single variable review!

7. Suppose $f : \mathbb{R} \to \mathbb{R}$ is differentiable, $f(0) = 0$, and f' is increasing. Prove that the function $g : (0, \infty) \to \mathbb{R}$ given by

$$g(x) = \frac{f(x)}{x}$$

is increasing.