1. True or False?

The set $S = \{(x, y) \in \mathbb{R}^2 : x \geq 0 \text{ or } y \neq 0\}$ is convex.

2. Suppose I is an interval and $f : I \to \mathbb{R}$ is concave down and $f \geq 0$. Prove that the set

$$S = \{(x, y) : x \in I \text{ and } 0 \leq y \leq f(x)\}$$

is a convex subset of $\mathbb{R}^2$.

3. True or False?

Suppose $f : \mathbb{R}^n \to \mathbb{R}^n$ is differentiable and injective. Then f is étale.

4. True or False?

Suppose $f : \mathbb{R}^n \to \mathbb{R}^n$ is differentiable, injective, and f^{-1} is also differentiable. Then f is étale.

5. True or False?

Suppose $f : \mathbb{R}^2 \to \mathbb{R}^2$ is differentiable and $f(0,0) = f(0,1)$. Then there exists a real number c between 0 and 1 such that $f'(0,c) = 0$.

6. True or False?

Let $U = \{(x, y) : x > 0\}$ and let $f : U \to \mathbb{R}^2$ be the function

$$f(x, y) = (xe^{-y}, xe^{y}).$$

Then f is étale.

6. True or False?

Let $U = \{(x, y) : x > 0\}$ and let $f : U \to \mathbb{R}^2$ be the function

$$f(x, y) = (xe^{-y}, xe^{y}).$$

Then f is étale.

Follow-up. What is $f(U)$?

7. True or False?

Let $U = \{(x, y) : x > 0\}$ and let $f : U \to \mathbb{R}^2$ be the function

$$f(x, y) = (xe^{-y}, xe^{y}).$$

Then f is injective.

8. With the same function f as in the previous problems, observe that $f(1,0) = (1,1)$. Calculate $f'(1,0)$ and $(f^{-1})'(1,1)$ and compare.