1. Let $\ell : \mathbb{R}^2 \to \mathbb{R}$ be the linear map

$$\ell(x, y) = x + y.$$

What is the max operator norm $\|\ell\|_\infty$ of ℓ?

(a) 1

(b) 2

(c) 3

(d) None of the above

2. Let $\ell : \mathbb{R}^2 \to \mathbb{R}$ be the linear map

$$\ell(x, y) = x + y.$$

What is the euclidean operator norm $\|\ell\|_2$ of ℓ?

(a) 1

(b) 2

(c) 3

(d) None of the above

3. True or False?

Suppose $\ell : \mathbb{R} \to \mathbb{R}^n$ is a linear map. Then

$$\|\ell\| = |\ell(1)|.$$

4. The level sets of the function $f : \mathbb{R}^2 \to \mathbb{R}$ given by

$$f(x, y) = y - x^2$$

are...

(a) Lines

(b) Parabolas

(c) Some are lines, some are parabolas

(d) None of the above

5. True or False?

If $f : \mathbb{R}^2 \to \mathbb{R}$ is given by

$$f(x, y) = 2x + 3y + x^2,$$

then $df_{(0,0)}(h, k) = 2h + 3k$.

6. True or False?

The euclidean norm function $f : \mathbb{R}^2 \to \mathbb{R}$ given by

$$f(x) = |x|_2$$

is differentiable at the origin.

7. Single variable review! Prove this:

L'Hôpital's rule (less weak version)

Suppose f, g are C^k functions for some $k \geq 1$ and

$$f(a) = f'(a) = \cdots = f^{(k-1)}(a) = 0$$
$$g(a) = g'(a) = \cdots = g^{(k-1)}(a) = 0$$

and $g^{(k)}(a) \neq 0$. Then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{f^{(k)}(a)}{g^{(k)}(a)}.$$