1. Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ given by

$$f(x, y) = \begin{cases} 1 & \text{if } x = 0 \text{ or } y = 0 \\ 0 & \text{otherwise.} \end{cases}$$

Then...

(A) f is differentiable at 0.

(B) f is not differentiable at 0, but the partial derivatives $\partial_1 f(0)$ and $\partial_2 f(0)$ exist.

(C) The partial derivatives $\partial_1 f(0)$ and $\partial_2 f(0)$ do not exist.

2. Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ given by

$$f(x, y) = 2x^3 - 3x^2 + 2y^3 + 3y^2.$$

How many critical points does f have?

(A) 2

(B) 4

(C) 6

(D) None of above

3. Consider the same function

$$f(x,y) = 2x^3 - 3x^2 + 2y^3 + 3y^2$$

as in the previous problem. How many local extremums does f have?

(A) 0

(B) 2

(C) 4

(D) None of above

4. True or False?

There exists a function $f : \mathbb{R}^2 \to \mathbb{R}$ which is differentiable at 0 such that

$$\partial_h f(0) > 0$$

for all $h \in \mathbb{R}^2$.

5. True or False?

Suppose $f : \mathbb{R}^n \to \mathbb{R}$ is differentiable at $a \in \mathbb{R}^n$. Then there exists $h \in \mathbb{R}^n$ such that $|h| = 1$ and

$$\|df_a\| = |\partial_h f(a)|.$$