For any real $r > 0$, let $h_r : \mathbb{R} \to \mathbb{R}$ be the function defined by

$$h_r(t) = \begin{cases} t & \text{if } t \leq 0 \\ rt & \text{if } t > 0 \end{cases}$$

and observe that h_r is a strictly increasing function, so $(\mathbb{R}, h_r)$ is a chart.

1. True or False?

If $r \neq s$ are positive real numbers, then $(\mathbb{R}, h_r)$ and $(\mathbb{R}, h_s)$ are incompatible charts.

Single variable review!

2. True or False?

There exist functions $f, g : \mathbb{R} \to \mathbb{R}$ such that f is surjective and differentiable, and $g \circ f$ is differentiable, but g is not differentiable.

Let $h : (0, 2\pi) \to S^1$ be the function given by

$$h(\theta) = (\cos\theta, \sin\theta)$$

and let $W = h((0, 2\pi)) = S^1 \setminus \{(1, 0)\}$. Then the pair (W, h^{-1}) is a chart.

3. True or False?

(W, h^{-1}) is compatible with the atlas on S^1 from last night (cf. exercise 3.2.4).

Single and multivariable review!

4. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be the function

$$f(x,y) = \begin{cases} \dfrac{x^3}{x^2+y^2} & \text{if } (x,y) \neq 0 \\ 0 & \text{if } (x,y) = 0. \end{cases}$$

Suppose that $\gamma : \mathbb{R} \to \mathbb{R}^2$ is differentiable, $\gamma(0) = 0$ and $\gamma'(0) \neq 0$. Prove that $f \circ \gamma$ is differentiable at 0.

Possible hint. Use the definition of differentiability for the single variable function $f \circ \gamma$. You might use the "error in approximation" functions for the component functions γ_1 and γ_2 of γ.