# Worksheet 8

**Problem 1.** Find absolute extrema for the following functions on the specified domains if they exist, and the $x$-values where they occur.

a) $f(x) = x^3 - 3x^2 - 24x + 5$ on $[-3, 6]$

b) $f(x) = (8 + x)/(8 - x)$ on $[4, 6]$

c) $f(x) = x/(x^2 + 2)$ on $[0, 4]$

d) $f(x) = (x^2 - 16)^{2/3}$ on $[-5, 8]$

e) $f(x) = \ln(x)/x^2$ on $[1, 4]$

f) $f(x) = x^2 e^{-x/2}$ on $[2, 5]$

g) $f(x) = 12 - x - (9/x)$ on $(0, \infty)$

h) $f(x) = x^4 - 4x^3 + 4x^2 + 1$ on $(-\infty, \infty)$

i) $f(x) = x/(x^2 + 1)$ on $(-\infty, \infty)$

j) $f(x) = x \ln(x)$ on $(-\infty, \infty)$

**Problem 2.** A company has found that its weekly profit from the sale of $x$ units of an auto part is given by

$$P(x) = -0.02x^3 + 600x - 20000.$$

Production bottlenecks limit the number of units that can be made per week to no more than 150, while a long-term company contract requires that at least 50 units be made each week. Find the maximum possible weekly profit that the firm can make.

**Problem 3.** The number of salmon swimming upstream to spawn is approximated by

$$S(x) = -x^3 + 3x^2 + 360x + 5000, \quad 6 \leq x \leq 20$$

where $x$ represents the temperature of the water in degrees Celsius. Find the water temperature that produces the maximum number of salmon swimming upstream.

**Problem 4.** The equation

$$M(x) = -(1/45)x^2 + 2x - 20, \quad 30 \leq x \leq 65$$

represents the miles per gallon used by a certain car at a speed of $x$ mph. Find the absolute maximum miles per gallon and the absolute minimum and the speeds at which they occur.

**Problem 5.** A piece of wire 12 ft long is cut into two pieces. One piece is made into a circle and the other into a square. Let $x$ denote the length of the piece that is made into a circle. We allow $x$ to equal 0 or 12, so that all of the wire may be used for the square or for the circle.

a) Where should the cut be made in order to minimize the sum of the areas enclosed by both figures?

b) Where should the cut be made in order to maximize the sum of the areas enclosed by both figures?

**Problem 6.** Find the dimensions of the rectangular field of maximum area that can be made from 300 m of fencing material.

**Problem 7.** A fence must be built to enclose a rectangular area of 20,000 ft$^2$. Fencing material costs \$2.50 per foot for two sides facing north and south, and \$3.20 per foot for the other two sides. Find the cost of the least expensive fence.