Worksheet 7

Problem 1. For each of the following, calculate the critical numbers, where the function is increasing, and where the function is decreasing.

a) $f(x) = (2/3)x^3 - x^2 - 4x + 2$

b) $f(x) = (x+3)/(x-4)$

c) $f(x) = xe^{x^2-3x}$

d) $f(x) = x^2 2^{-x}$

e) $f(x) = \sqrt{x^2+1}$

f) $f(x) = x - 4\ln(3x-9)$

Problem 2. The total cost $C(x)$ in dollars to manufacture quantity x of weed killer (in hundreds of liters) is given by $C(x) = x^3 - 2x^2 + 8x + 50$. Where is $C(x)$ increasing? Where is it decreasing?

Problem 3. The percent concentration of a drug in the bloodstream t hours after the drug is administered is given by

$$K(t) = \frac{4t}{3t^2+27}$$

for $t \geq 0$. On what intervals is drug concentration increasing? On what intervals is it decreasing? When is drug concentration at its maximum?

Problem 4. The demand equation for telephones at one store is

$$p = D(q) = 200e^{-q/10}$$

where p is the price in dollars and q is the quantity of telephones sold per week. Find the values of q and p that maximize the revenue.

Problem 5. When a bottle of champagne is shaken several times, held upright, and uncorked, its cork travels according to

$$s(t) = -16t^2 + 40t + 3$$

where s is the height in feet above the ground t seconds after being released. How high will the cork go? How long is it in the air?

Problem 6. Find the x-value of all points where the following functions have relative extrema. Then determine if the relative extremum is a relative maximum or minimum, and calculate the value of the function at that point.

a) $f(x) = (x^2 - 6x + 9)/(x+2)$

b) $f(x) = 3xe^x + 2$

c) $f(x) = 2x + \ln(x)$

d) $f(x) = 2^x/x$

Problem 7. Find $f''(x)$ for each of the following functions. Find where the function is concave up and where it is concave down.

a) $f(x) = x^2 + 10x - 9$

b) $f(x) = 3/(x-5)$

c) $f(x) = x(x+5)^2$

d) $f(x) = 2e^{-x^2}$

Problem 8. Let $f(t)$ denote the rate of violent crime in New York City at time t. If you're told that in 1995, the rate of violent crimes continued to increase, but at a slower rate than in previous years, what does this tell you about $f(t), f'(t)$ and $f''(t)$?