Worksheet 5

Problem 1. Find derivatives of each of the following functions.

a) $f(t) = 8t^3 - 5t^2 - t/12$

b) $f(x) = -2x^{1.5} + 12x^{0.5}$

c) $f(t) = 7/t - 5/t^3$

d) $f(x) = (x^3 + 5)/x$

e) $f(t) = (3t^2 + 2)^2$

f) $f(x) = (8x - 11)/(7x + 3)$

g) $f(t) = \sqrt{t}/(2t + 3)$

h) $f(x) = -6x(5x^4 - 1)^4$

i) $f(t) = -3\sqrt{7t^3 - 1}$

j) $f(x) = -5/(2x^3 + 1)^2$

Problem 2. Find the equation of the tangent line to $y = x^4 - 5x^3 + 2$ at $x = 2$.

Problem 3. Find all points where the tangent line to the function $f(x) = x^3 + 15x^2 + 63x - 10$ is horizontal.

Problem 4. Suppose g and h are functions such that $g'(5) = 12$ and $h'(5) = -3$. Calculate $f'(5)$, where $f(x) = 3g(x) - 2h(x) + 3$.

Problem 5. A company's costs and revenues in dollars for a product are given by $C(x) = 2x$ and $R(x) = 6x - x^2/1000$, respectively, where x is the number of items produced.

a) Find the marginal cost function.

b) Find the marginal revenue function.

c) Find the marginal profit function.

d) What value of x makes the marginal profit equal to 0?

e) Find the profit when the marginal profit is 0.

Problem 6. A company that manufactures bicycles has determined that a new employee can assemble $M(d)$ bicycles after d days of on-the-job training, where $M(d) = 100d^2/(3d^2 + 10)$.

a) Find the rate of change for the number of bicycles assembled with respect to time.

b) Find and interpret $M'(2)$ and $M'(5)$.

Problem 7. The "Pythagorean Theorem of Baseball" says that

$$W = \frac{s^2}{s^2 + a^2},$$

where s is the number of runs a team scores, a is the number of runs a team allows, and W is a predictor for the percentage of games a team is expected to win.

In 2013, the team with the best record was the Boston Red Sox, with 97 wins and 67 losses. They scored 853 runs and allowed 656 runs.

a) Calculate the actual winning percentage, and the percentage predicted by the Pythagorean Theorem of Baseball.

b) Keeping the runs scored fixed at $s = 853$ and considering the runs allowed as a variable a, find dW/da when $a = 656$.

c) Interpreting your answer to part b) as the approximate amount that W would change if a increased by one run, approximate W when $a = 657$.

d) Compare your answer to part c) with what the actual value of W would be if $s = 853$ and $a = 657$.