Worksheet 5

Problem 1. Find derivatives of each of the following functions.

- a) $f(t) = 8t^3 5t^2 t/12$ b) $f(x) = -2x^{1.5} + 12x^{0.5}$ c) $f(t) = 7/t - 5/t^3$ d) $f(x) = (x^3 + 5)/x$ e) $f(t) = (3t^2 + 2)^2$ f) f(x) = (8x - 11)/(7x + 3)h) $f(x) = -6x(5x^4 - 1)^4$ i) $f(t) = -3\sqrt{7t^3 - 1}$ j) $f(x) = -5/(2x^3 + 1)^2$ Answer. a) $f'(t) = 24t^2 - 10t - 1/12$ b) $f'(x) = -3x^{0.5} + 6x^{-0.5}$ c) $f'(t) = -7/t^2 + 15/t^4$ d) $f'(x) = 2x - 5/x^2$
- e) $f'(t) = 12t(3t^2 + 2)$ f) $f'(x) = 101/(7x + 3)^2$ g) $f'(t) = (3 - 2t)/2\sqrt{t}(2t + 3)^2$ h) $f'(x) = 6(1 - 85x^4)(5x^4 - 1)^3$ j) $f'(x) = 60x^2/(2x^3 + 1)^3$

Problem 2. Find the equation of the tangent line to $y = x^4 - 5x^3 + 2$ at x = 2.

Answer. y = 34 - 28x

Problem 3. Find all points where the tangent line to the function $f(x) = x^3 + 15x^2 + 63x - 10$ is horizontal. Answer. f'(x) = 3(x+3)(x+7) so the tangent line is horizontal at x = -3 and x = -7.

Problem 4. Suppose g and h are functions such that g'(5) = 12 and h'(5) = -3. Calculate f'(5), where f(x) = 3g(x) - 2h(x) + 3.

Answer. f'(5) = 3g'(5) - 2h'(5) = 36 + 6 = 42

Problem 5. A company's costs and revenues in dollars for a product are given by C(x) = 2x and $R(x) = 6x - x^2/1000$, respectively, where x is the number of items produced.

- a) Find the marginal cost function.
- b) Find the marginal revenue function.
- c) Find the marginal profit function.
- d) What value of x makes the marginal profit equal to 0?
- e) Find the profit when the marginal profit is 0.

Answer.

- a) C'(x) = 2
- b) R'(x) = 6 x/500

- c) $P(x) = 4x x^2/1000$ and P'(x) = 4 x/500
- d) P'(x) = 0 when x = 2000
- e) P(2000) = 4000

Problem 6. A company that manufactures bicycles has determined that a new employee can assemble M(d) bicycles after d days of on-the-job training, where $M(d) = 100d^2/(3d^2 + 10)$.

- a) Find the rate of change for the number of bicycles assembled with respect to time.
- b) Find and interpret M'(2) and M'(5).

Answer.

- a) $M'(d) = 2000d/(3d^2 + 10)^2$
- b) M'(2) = 1000/121 is roughly the number of additional bikes produced by an employee after an additional day of on-the-job draining after the employee has already had 2 days of training. M'(5) = 400/289 has a similar interpretation.

Problem 7. The "Pythagorean Theorem of Baseball" says that

$$W = \frac{s^2}{s^2 + a^2},$$

where s is the number of runs a team scores, a is the number of runs a team allows, and W is a predictor for the percentage of games a team is expected to win.

In 2013, the team with the best record was the Boston Red Sox, with 97 wins and 67 losses. They scored 853 runs and allowed 656 runs.

- a) Calculate the actual winning percentage, and the percentage predicted by the Pythagorean Theorem of Baseball.
- b) Keeping the runs scored fixed at s = 853 and considering the runs allowed as a variable a, find dW/da when a = 656.
- c) Interpreting your answer to part b) as the approximate amount that W would change if a increased by one run, approximate W when a = 657.
- d) Compare your answer to part c) with what the actual value of W would be if s = 853 and a = 657.

Answer.

- a) The real winning percentage is 97/164 \approx 0.591. The percentage prediced by the Pythagorean Theorem is $W \approx 0.6283$.
- b) $(dW/da)|_{a=656} = -0.00071$
- c) When a = 657, we have $W \approx 0.6283 + (dW/da)|_{a=656} = 0.62765$
- d) W is actually $853^2/(853^2+657^2) \approx 0.62765$