Worksheet 3

Problem 1. Calculate the following limits, if they exist. If they do not exist, determine if they are $\pm\infty$.

a) $\lim_{x\to -5} \dfrac{x^2-25}{x+5}$

b) $\lim_{x\to -3} \dfrac{x^2-9}{x^2+x-6}$

c) $\lim_{x\to -\infty} \dfrac{8x+2}{4x-5}$

d) $\lim_{x\to \infty} \dfrac{x^2+2x-5}{3x^2+2}$

e) $\lim_{x\to \infty} \dfrac{2x^3-x-3}{6x^2-x-1}$

f) $\lim_{x\to -\infty} \dfrac{-5x^3-4x^2+8}{6x^2+3x+2}$

g) $\lim_{x\to 0} \dfrac{|x|}{x}$

h) $\lim_{x\to 3} \dfrac{x}{x^2-3x}$

i) $\lim_{x\to 3} \dfrac{\sqrt{x}(x-3)}{x^2-6x+9}$

j) $\lim_{x\to \infty} \big(\ln(2x^3+1) - 3\ln(x-1)\big)$

k) $\lim_{x\to -1} e^{\left(\dfrac{x}{x^3+2x^2+x}\right)}$

l) $\lim_{x\to 2} \dfrac{x^2-4x+4}{(x-2)^3}$

Problem 2. A company training program has determined that, on average, a new employee produces $P(s)$ items per day after s days of on-the-job training, where

$$P(s) = \frac{63s}{s+8}.$$

Calculate $\lim_{s\to\infty} P(s)$, and interpret what this value means in words.

Problem 3. Consider the function $f(x) = \ln\left|\dfrac{x+2}{x-3}\right|$.

a) Explain why the domain of f is all real numbers except -2 and 3.

b) Calculate $\lim_{x\to -2^-} f(x)$ and $\lim_{x\to -2^+} f(x)$.

c) Calculate $\lim_{x\to 3^-} f(x)$ and $\lim_{x\to 3^+} f(x)$.

Problem 4. Sketch graphs of each of the following functions. At what points, if any, are each of them discontinuous?

a) $f(x) = \dfrac{|x+2|}{x+2}$

b) $f(x) = \begin{cases} x-1 & \text{if } x<1 \\ 0 & \text{if } 1\le x\le 4 \\ x-2 & \text{if } x>4 \end{cases}$

Problem 5. Find the value of the constant k that makes the following function continuous.

$$f(x) = \begin{cases} x^3+k & \text{if } x\le 3 \\ kx-5 & \text{if } x>3 \end{cases}$$