Worksheet 3

Problem 1. Calculate the following limits, if they exist. If they do not exist, determine if they are $\pm\infty$.

a) $\lim_{x\to -5} \dfrac{x^2-25}{x+5}$

b) $\lim_{x\to -3} \dfrac{x^2-9}{x^2+x-6}$

c) $\lim_{x\to -\infty} \dfrac{8x+2}{4x-5}$

d) $\lim_{x\to \infty} \dfrac{x^2+2x-5}{3x^2+2}$

e) $\lim_{x\to \infty} \dfrac{2x^3-x-3}{6x^2-x-1}$

f) $\lim_{x\to -\infty} \dfrac{-5x^3-4x^2+8}{6x^2+3x+2}$

g) $\lim_{x\to 0} \dfrac{|x|}{x}$

h) $\lim_{x\to 3} \dfrac{x}{x^2-3x}$

i) $\lim_{x\to 3} \dfrac{\sqrt{x}(x-3)}{x^2-6x+9}$

j) $\lim_{x\to \infty} \big(\ln(2x^3+1) - 3\ln(x-1)\big)$

k) $\lim_{x\to -1} e^{\left(\dfrac{x}{x^3+2x^2+x}\right)}$

l) $\lim_{x\to 2} \dfrac{x^2-4x+4}{(x-2)^3}$

Answer.

a) -10

b) $6/5$

c) 2

d) $1/3$

e) ∞

f) ∞

g) DNE

h) DNE

i) DNE

j) $\ln(2)$

k) ∞

l) DNE

Problem 2. A company training program has determined that, on average, a new employee produces $P(s)$ items per day after s days of on-the-job training, where

$$P(s) = \frac{63s}{s+8}.$$

Calculate $\lim_{s\to\infty} P(s)$, and interpret what this value means in words.

Answer. The limit is 63. This is the average number of items produced by an employee per day after many many days of on-the-job training.

Problem 3. Consider the function $f(x) = \ln\left|\dfrac{x+2}{x-3}\right|$.

a) Explain why the domain of f is all real numbers except -2 and 3.

b) Calculate $\lim_{x\to -2^-} f(x)$ and $\lim_{x\to -2^+} f(x)$.

c) Calculate $\lim_{x\to 3^-} f(x)$ and $\lim_{x\to 3^+} f(x)$.

Answer.

a) Logarithms are defined on all *strictly positive* real numbers. The expression inside log is an absolute value, so it is always nonnegative when it is defined. It equals 0 when $x = -2$, so the function is not defined there, and the expression isn't defined at all when $x = 3$.

b) $-\infty$

c) ∞

Problem 4. Sketch graphs of each of the following functions. At what points, if any, are each of them discontinuous?

a) $f(x) = \dfrac{|x+2|}{x+2}$

b) $f(x) = \begin{cases} x-1 & \text{if } x < 1 \\ 0 & \text{if } 1 \le x \le 4 \\ x-2 & \text{if } x > 4 \end{cases}$

Answer. Omitted.

Problem 5. Find the value of the constant k that makes the following function continuous.

$$f(x) = \begin{cases} x^3 + k & \text{if } x \le 3 \\ kx - 5 & \text{if } x > 3 \end{cases}$$

Answer. $k = 16$