

WORKSHEET 2

Problem 1. Calculate the following.

(a) $\log_3(81)$

(b) $\log_8 \sqrt[4]{\frac{1}{2}}$

Problem 2. Solve the following equations.

(a) $\left(\frac{1}{27}\right)^{2x} = 9^{x^2-3x-1}$.

(b) $\log_5(9x - 4) = 1$.

Problem 3. Calculate the compound amount when \$18,000 is invested with 12% interest compounded monthly over 2 years.

Problem 4. A Petri dish starts with a population of 500 bacteria, and the population doubles every half hour.

(a) Find an expression for the population $P(t)$ of bacteria in the Petri dish after t hours have elapsed.

(b) After how many hours are there 16,000 bacteria in the Petri dish?

Problem 5. The number of years $N(r)$ since two independently evolving languages split off from a common ancestral language is approximated by

$$N(r) = -5000 \ln r$$

where r is the proportion of words from the ancestral language that are common to both languages now.

(a) How many years have elapsed if 70% of the words of the ancestral language are common to both languages today?

(b) If two languages split off from a common ancestral language about 1000 years ago, find r .

Problem 6. Suppose

$$\lim_{x \rightarrow 4} f(x) = 15 \text{ and } \lim_{x \rightarrow 4} g(x) = 25.$$

Calculate each of the following.

(a) $\lim_{x \rightarrow 4} (f(x) - g(x))$

(b) $\lim_{x \rightarrow 4} \sqrt{g(x)}$

(c) $\lim_{x \rightarrow 4} \frac{f(x) + g(x)}{2g(x)}$

Problem 7. Calculate each of the following limits.

(a) $\lim_{x \rightarrow -2} \frac{x^2 - x - 6}{x + 2}$

(b) $\lim_{x \rightarrow 36} \frac{x - 36}{\sqrt{x} - 6}$

(c) $\lim_{x \rightarrow 1} f(x)$, where $f(x) = \begin{cases} x^2 + 2 & \text{if } x \neq 1 \\ 17 & \text{if } x = 1 \end{cases}$

(d) $\lim_{x \rightarrow 2} f(x)$, where $f(x) = \begin{cases} x^2 & \text{if } x < 2 \\ 5 & \text{if } x = 2 \\ 6 - x & \text{if } x > 2 \end{cases}$