Name:

Quiz 6

You must show all of your work for full credit.

Problem 1 (5 points)**.** Calculate $\displaystyle\int \frac{10x}{3+5x^2}\,dx$.

Solution. We use substitution: let $u = 3+5x^2$, so that $du = 10x\,dx$. Then

$$\int \frac{10x}{3+5x^2}\,dx = \int \frac{du}{u} = \ln|u| + C = \ln|3+5x^2| + C = \ln(3+5x^2) + C,$$

where the final equality is because $3+5x^2 \geq 0$ for all x.

Problem 2 (5 points)**.** Calculate $\displaystyle\int x(x+1)^5\,dx$.

Solution. We use substitution: let $u = x+1$, so that $du = dx$. Note that we have $x = u-1$, so

$$\int x(x+1)^5\,dx = \int (u-1)u^5\,du = \int (u^6 - u^5)\,du = \frac{u^7}{7} - \frac{u^6}{6} + C = \frac{(x+1)^7}{7} - \frac{(x+1)^6}{6} + C.$$

Problem 3 (5 points)**.** Use $n = 4$ rectangles and right endpoints to approximate the area under the curve $y = x^2$ between $x = 1$ and $x = 5$. Is this approximation an underestimate or an overestimate for the actual area?

Solution. Dividing $[1, 5]$ into 4 intervals, we get $[1, 2], [2, 3], [3, 4]$, and $[4, 5]$. Each rectangle has width 1, and their heights are $4, 9, 16$, and 25. Thus the area is approximately

$$4 + 9 + 16 + 25 = 54.$$

This is an overestimate for the actual area.

Problem 4 (5 points)**.** Use formulas from geometry to calculate $\int_1^3 (5 + x)\, dx$.

Solution. We are trying to calculate the area of a trapezoid, which we can break up into two pieces: a rectangle of width 2 and height 6, and then a right triangle of width 2 and height 2. These have areas 12 and 2, respectively, so

$$\int_1^3 (5 + x)\, dx = 14.$$