Name:

Quiz 5

You must show all of your work for full credit.

Problem 1 (5 points). Find the slope of the tangent line to the curve $x^2y^3 = 8$ at the point (-1, 2).

Solution. We have $2xy^3 + 3x^2y^2(dy/dx) = 0$, so

$$\frac{dy}{dx} = -\frac{2xy^3}{3x^2y^2} = -\frac{2y}{3x}.$$

Thus the slope of the tangent line at (-1,2) is

$$\left. \frac{dy}{dx} \right|_{(x,y)=(-1,2)} = \frac{4}{3}.$$

Problem 2 (5 points). Calculate $\int \frac{6t^3 + 4t}{2t} dt$.

Solution.

$$\int \frac{6t^3 + 4t}{2t} = \int (3t^2 + 2) = t^3 + 2t + C.$$

Problem 3 (5 points). Calculate $\int (e^{u/2} + 4u) du$.

Solution.

$$\int (e^{u/2} + 4u) \, du = 2e^{u/2} + 2u^2 + C.$$

Problem 4 (5 points). The marginal profit of a small fast-food stand in thousands of dollars is given by

$$P'(x) = \sqrt{x} + \frac{1}{2}$$

where x the the sale volume in thousands of hamburgers. The "profit" is -\$1000 when no hamburgers are sold. Find the profit function.

Solution. We have

$$P(x) = \int \left(\sqrt{x} + (1/2)\right) = \frac{2x^{3/2}}{3} + \frac{x}{2} + C$$

for some constant C, and P(0) = C = -1. Thus

$$P(x) = \frac{2x^{3/2}}{3} + \frac{x}{2} - 1.$$