

Name:

### QUIZ 4

You must show all of your work for full credit.

**Problem 1** (5 points). A phone manufacturer has determined that the profit  $P(x)$  in thousands dollars is related to the quantity  $x$  of phones produced (in hundreds) per month by

$$P(x) = -(x - 4)e^x - 4.$$

At what production levels is the profit increasing? At what levels is it decreasing?

*Solution.*  $P'(x) = -e^x - (x - 4)e^x = -e^x(x - 3)$ , so  $x = 3$  is the only critical number of  $P$ .  $P'(x)$  is positive when  $x < 3$  and negative when  $x > 3$ , so profit is increasing when fewer than 300 phones are produced and decreasing when more than 300 are produced.

**Problem 2** (5 points). A group of researchers has found that people prefer training films of moderate length; shorter films contain too little information, while longer films are boring. For a training film on the care of exotic birds, the researchers determined that the ratings people gave for the film could be approximated by

$$R(t) = \frac{20t}{t^2 + 100},$$

where  $t$  is the length of the film in minutes. Find the film length that received the highest rating.

*Solution.* Using the quotient rule,

$$R'(t) = \frac{20(t^2 + 100) - 20t(2t)}{(t^2 + 100)^2} = \frac{-20t^2 + 2000}{(t^2 + 100)^2} = \frac{-20(t^2 - 100)}{(t^2 + 100)^2}.$$

so  $R'(t) = 0$  when  $t = \pm 10$ . Negative values of  $t$  don't make sense. When  $t < 10$ , notice that  $R'(t)$  is positive, and it is negative when  $t > 10$ . Thus 10 is a maximum of  $R$  on the domain  $[0, \infty)$ . In other words, 10-minute films received the best rating.

**Problem 3** (10 points). Consider the function  $f(x) = x^4 - 20x^2 + 64$ .

- (a) What are the  $x$ -intercepts of  $f$ ? What is the  $y$ -intercept?
- (b) What are the the critical points of  $f$ ? On what intervals is  $f$  increasing? Decreasing?
- (c) What are the  $x$ -values of the inflection points of  $f$ ? On what intervals is  $f$  concave up? Concave down?
- (d) Sketch a graph of  $f$ .

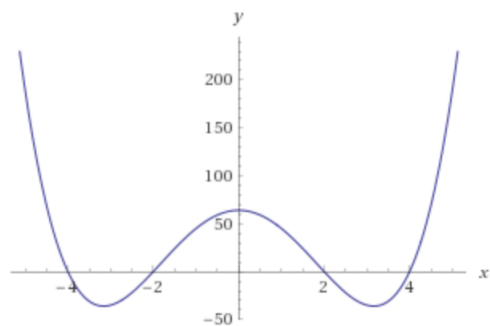
*Solution.*

(a)  $f(x) = x^4 - 20x^2 + 64 = (x^2 - 16)(x^2 - 4) = (x + 4)(x - 4)(x + 2)(x - 2)$ , so the  $x$ -intercepts of  $f$  are at  $\pm 2$  and  $\pm 4$ . The  $y$ -intercept is  $f(0) = 64$ .

(b)  $f'(x) = 4x^3 - 40x = 4x(x^2 - 10)$  so the critical numbers are at  $x = 0$  and  $x = \pm\sqrt{10}$ . The  $y$ -values at these points are  $f(-\sqrt{10}) = f(\sqrt{10}) = 100 - 200 + 64 = -32$  and  $f(0) = 64$ .

Notice that  $x^2 - 10$  is negative when  $|x| < \sqrt{10}$  and positive when  $|x| > \sqrt{10}$ . Thus  $f'$  is positive on  $(-\sqrt{10}, 0) \cup (\sqrt{10}, \infty)$  and negative everywhere else.

(c)  $f''(x) = 12x^2 - 40 = 12(x^2 - 10/3)$ , so the inflection points are at  $x = \pm\sqrt{10/3}$ . Thus  $f$  is concave down on  $(-\sqrt{10/3}, \sqrt{10/3})$  and concave up everywhere else.



(d)