Name:

Quiz 4

You must show all of your work for full credit.

Problem 1 (5 points)**.** A phone manufacturer has determined that the profit $P(x)$ in thousands dollars is related to the quantity x of phones produced (in hundreds) per month by

$$P(x) = -(x-4)e^x - 4.$$

At what production levels is the profit increasing? At what levels is it decreasing?

Solution. $P'(x) = -e^x - (x-4)e^x = -e^x(x-3)$, so $x = 3$ is the only critical number of P. $P'(x)$ is positive when $x < 3$ and negative when $x > 3$, so profit is increasing when fewer than 300 phones are produced and decreasing when more than 300 are produced.

Problem 2 (5 points)**.** A group of researchers has found that people prefer training films of moderate length; shorter films contain too little information, while longer films are boring. For a training film on the care of exotic birds, the researchers determined that the ratings people gave for the film could be approximated by

$$R(t) = \frac{20t}{t^2+100},$$

where t is the length of the film in minutes. Find the film length that received the highest rating.

Solution. Using the quotient rule,

$$R'(t) = \frac{20(t^2+100) - 20t(2t)}{(t^2+100)^2} = \frac{-20t^2+2000}{(t^2+100)^2} = \frac{-20(t^2-100)}{(t^2+100)^2}.$$

so $R'(t) = 0$ when $t = \pm 10$. Negative values of t don't make sense. When $t < 10$, notice that $R'(t)$ is positive, and it is negative when $t > 10$. Thus 10 is a maximum of R on the domain $[0, \infty)$. In other words, 10-minute films received the best rating.

Problem 3 (10 points)**.** Consider the function $f(x) = x^4 - 20x^2 + 64$.

(a) What are the x-intercepts of f? What is the y-intercept?

(b) What are the the critical points of f? On what intervals is f increasing? Decreasing?

(c) What are the x-values of the inflection points of f? On what intervals is f concave up? Concave down?

(d) Sketch a graph of f.

Solution.

(a) $f(x) = x^4 - 20x^2 + 64 = (x^2 - 16)(x^2 - 4) = (x+4)(x-4)(x+2)(x-2)$, so the x-intercepts of f are at ± 2 and ± 4. The y-intercept is $f(0) = 64$.

(b) $f'(x) = 4x^3 - 40x = 4x(x^2 - 10)$ so the critical numbers are at $x = 0$ and $x = \pm\sqrt{10}$. The y-values at these points are $f(-\sqrt{10}) = f(\sqrt{10}) = 100 - 200 + 64 = -32$ and $f(0) = 64$.

Notice that $x^2 - 10$ is negative when $|x| < \sqrt{10}$ and positive when $|x| > \sqrt{10}$. Thus f' is positive on $(-\sqrt{10}, 0) \cup (\sqrt{10}, \infty)$ and negative everywhere else.

(c) $f''(x) = 12x^2 - 40 = 12(x^2 - 10/3)$, so the inflection points are at $x = \pm\sqrt{10/3}$. Thus f is concave down on $(-\sqrt{10/3}, \sqrt{10/3})$ and concave up everywhere else.

(d)