Name:

Quiz 3

You must show all of your work for full credit.

Problem 1. Suppose $f(x) = (x+1)^4(x^3+4)$. Calculate $f'(0)$.

Solution. We use the product rule and then the chain rule.

$$\begin{aligned} f'(x) &= ((x+1)^4)'(x^3+4) + (x+1)^4(x^3+4)' \\ &= 4(x+1)^3(x^3+4) + 3x^2(x+1)^4 \end{aligned}$$

Plugging in $x = 0$, we get

$$f'(0) = 4 \cdot 1^3 \cdot 4 = 16.$$

Problem 2. Suppose $f(x) = \dfrac{(3x^2+1)(2x-1)}{5x+4}$. Calculate $f'(0)$.

Solution. We use the quotient rule and the product rule.

$$\begin{aligned} f'(x) &= \frac{((3x^2+1)(2x-1)')(5x+4) - 5(3x^2+1)(2x-1)}{(5x+4)^2} \\ &= \frac{(6x(2x-1) + 2(3x^2+1))(5x+4) - 5(3x^2+1)(2x-1)}{(5x+4)^2} \end{aligned}$$

Plugging in $x = 0$, we get

$$f'(0) = \frac{2 \cdot 1 \cdot 4 - 5 \cdot 1 \cdot (-1)}{16} = \frac{13}{16}.$$

Problem 3. A company's cost and revenue in dollars are given by

$$C(x) = 2x \text{ and } R(x) = 6x - \frac{x^2}{1000},$$

respectively, where x is the number of items produced. Find the value of x where the marginal *profit* is 0.

Solution. We have $C'(x) = 2$ and $R'(x) = 6 - x/500$, so if $P(x) = R(x) - C(x)$ is the profit function, then

$$P'(x) = R'(x) - C'(x) = 4 - x/500.$$

Solving $P'(x) = 0$ yields $x = 2000$.

Problem 4. The demand function for an item is given by

$$q = 30\left(5 - \frac{p}{\sqrt{p^2+1}}\right),$$

where p is the price in dollars and q is the quantity demanded. Find the rate of change in the demand for the product per unit change in price when $p = 3$.

Solution. We have

$$\frac{dq}{dp} = -30 \cdot \frac{\sqrt{p^2+1} - p^2(p^2+1)^{-1/2}}{p^2+1} = -\frac{30}{(p^2+1)^{3/2}},$$

so

$$\left.\frac{dp}{dq}\right|_{p=3} = -\frac{30}{10\sqrt{10}} = -\frac{3}{\sqrt{10}}.$$