Name:

Quiz 2

You must show all of your work for full credit.

Problem 1. Does

$$\lim_{x \to 3} \frac{x^2 - 9}{x - 3}$$

exist? If so, find its value. If it doesn't exist, does it equal $\pm\infty$? Explain your answer.

Solution.

$$\lim_{x \to 3} \frac{x^2 - 9}{x - 3} = \lim_{x \to 3} \frac{(x-3)(x+3)}{x - 3} = \lim_{x \to 3}(x+3) = 6.$$

Problem 2. Does

$$\lim_{x \to -\infty} \frac{2x^4 - 1}{3x^2 + 2},$$

exist? If so, find its value. If it doesn't exist, does it equal $\pm\infty$? Explain your answer.

Solution. The numerator is of higher degree, so this limit does not exist: it is either $\pm\infty$. Notice that as $|x|$ gets large, both the numerator and denominator are large positive numbers (since x^2 and x^4 are positive no matter whether x is positive or negative), so

$$\lim_{x \to -\infty} \frac{2x^4 - 1}{3x^2 + 2} = \infty.$$

Problem 3. Is there a value of k such that the following limit exists?

$$\lim_{x\to 3} \frac{2x^2+kx-3}{x^2-4x+3}$$

If so, find the value of k and compute the corresponding limit. Otherwise, explain why not.

Solution. The denominator factors as $(x-3)(x-1)$, so it vanishes at 3. The numerator equals $18+3k-3 = 15+3k$, so the limit can only possibly exist if $k=-5$. When $k=-5$, the numerator factors as $2x^2-5x-3 = (x-3)(2x+1)$, so

$$\lim_{x\to 3} \frac{2x^2-5x-3}{x^2-4x+3} = \lim_{x\to 3} \frac{(x-3)(2x+1)}{(x-3)(x-1)} = \lim_{x\to 3} \frac{2x+1}{x-1} = \frac{7}{2}.$$

Problem 4. The profit (in hundreds of dollars) from production of x units of an item is

$$P(x) = x^2-5x+6.$$

Using the limit definition of instantaneous rate of change, find the instantaneous rate of change of profit with respect to the number of items produced when $x=2$. Then write a sentence explaining how to interpret this quantity.

Solution. The instantaneous rate of change is

$$\begin{aligned}
\lim_{x\to 2} \frac{P(x)-P(2)}{x-2} &= \lim_{x\to 2} \frac{(x^2-5x+6)-(4-10+6)}{x-2} \\
&= \lim_{x\to 2} \frac{x^2-5x+6}{x-2} \\
&= \lim_{x\to 2} \frac{(x-2)(x-3)}{x-2} \\
&= \lim_{x\to 2} (x-3) \\
&= -1.
\end{aligned}$$

This means that when 2 units are produced, the profit is decreasing at a rate of \$100 for every additional unit produced.