

Name:

QUIZ 1

You must show all of your work for full credit.

Problem 1 (5 points). Find an equation of the line passing through $(1, 1)$ and perpendicular to $x + y = 4$.

Solution. The line $x + y = 4$ in slope-intercept form is

$$y = -x + 4$$

so it has slope -1 , so the slope of any perpendicular line must be 1 . Therefore the perpendicular line passing through $(1, 1)$ is

$$y - 1 = 1 \cdot (x - 1)$$

in point-slope form, or

$$y = x$$

in slope-intercept form.

Problem 2 (5 points). Sketch graphs of the functions $f(x) = \sqrt{x}$ and $g(x) = \sqrt{x - 2} + 2$.

Solution. Pictures omitted. The graph of $g(x)$ is the same as the graph of $f(x)$, except that it is shifted up 2 units and to the right 2 units.

Problem 3 (5 points). Suppose the supply and demand for a certain widget are given by

$$p = S(q) = 4q + 10 \text{ and } p = D(q) = 50 - 4q.$$

Find the equilibrium quantity and price for this widget.

Solution. Equilibrium happens when the supply and demand curves intersect. We set $4q + 10 = 50 - 4q$ and solve for q to get $8q = 40$ or $q = 5$. Thus the equilibrium quantity is $q = 5$ and the price is $S(5) = 4 \cdot 5 + 10 = 30$.

Problem 4 (5 points). The manager of a peach farm is trying to decide when to arrange for picking the peaches. If they are picked now, the average yield per tree will be 100 lb, which can be sold for 48¢ per pound. The yield will increase about 5 lb per week, while the price will decrease about 2¢ per week. When should the peaches be picked in order to maximize revenue?

Solution. If the peaches are harvested x weeks from now, the revenue $R(x)$ will be the yield (in pounds) per tree in x weeks multiplied by the price per pound of peaches in x weeks. The yield per tree in x weeks will be $100 + 5x$. The price per pound in x weeks will be $48 - 2x$. Thus

$$R(x) = (100 + 5x)(48 - 2x).$$

Graphing this, we get a downwards-facing parabola whose x -intercepts happen when $100 + 5x = 0$ and when $48 - 2x = 0$. The former is equivalent to $x = -20$, the latter to $x = 24$. The maximum revenue happens at the midpoint between these values, which is

$$\frac{-20 + 24}{2} = \frac{4}{2} = 2.$$

The peaches should be picked 2 weeks from now to maximize revenue.