# Errata 

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From category $\mathcal{O}^{\infty}$ to locally analytic representations (doi: 10.1016/j.jalgebra.2021.09.032, arXiv: 2011.12370v5)
We would like to extend our thanks for Zichuan Wang for a careful read-through of the paper.

- Section 2.4, line 3: $\mathfrak{g}=\bigoplus_{\alpha \in \mathfrak{t}^{*}} \mathfrak{g}_{\alpha}$ (no subscript $E$ is needed).
- Lemma 2.4.2: $\mathfrak{p}$ should be $\mathfrak{g}$.
- Equation 2.4.3: Right-hand side should be $\phi_{\mathrm{n}}(\mathrm{y})(\mathrm{x} . \mathrm{m})$, and in the following line, $\mathfrak{p}_{\alpha, \mathrm{E}}$ should be $\mathfrak{g}_{\alpha}$.
- Paragraph 2.4.6: $\mathfrak{p}$ should be $\mathfrak{g}$.
- Paragraph 2.5.3: The sixth sentence should end with $\left.\sigma\right|_{\mathrm{T}^{\prime}}=\left.\rho\right|_{\mathrm{T}^{\prime}}$ (one of the restriction bars was missing).
- Equation 2.5.4: Should be $\sigma\left(\operatorname{th}^{-1}\right)=\rho(t) \circ \sigma(h) \circ \rho\left(t^{-1}\right)$.
- Proof of equation 2.5.4: Replace the last four sentences with the following:

Next, suppose $h$ is in a root subgroup $U_{\alpha}$ of $P^{\prime}$. As we noted at the end of paragraph 2.5.2, the action of $h$ on $M$ is then given by

$$
\sigma(h)=\exp (\phi(\log (h)))
$$

Let $x=\log (h) \in \mathfrak{p}_{\alpha}$. Then tht $^{-1}=\exp \left(\chi_{\alpha}(t) x\right)$, so

$$
\sigma\left(\operatorname{th}^{-1}\right)=\exp \left(\phi\left(\chi_{\alpha}(t) x\right)\right)=\exp \left(\chi_{\alpha}(t) \phi(x)\right)
$$

On the other hand, we have

$$
\rho(t) \circ \sigma(h) \circ \rho(t)^{-1}=\rho(t) \circ \exp (\phi(x)) \circ \rho\left(t^{-1}\right)=\exp \left(\rho(t) \circ \phi(x) \circ \rho\left(t^{-1}\right)\right)
$$

To show that these two endomorphisms of $M$ coincide, it is sufficient to show that they coincide for any generalized weight vector $m \in M_{\lambda}$. Then $\rho\left(t^{-1}\right)(m) \in M_{\lambda}$, so $\left(\phi(x) \circ \rho\left(t^{-1}\right)\right)(m) \in M_{\alpha+\lambda}$ by lemma 2.4.1. Letting $y=\log (t)$, we see that

$$
\begin{aligned}
\left(\rho(t) \circ \phi(x) \circ \rho\left(t^{-1}\right)\right)(m) & =\left(\chi_{\alpha+\lambda}(t) \exp \left(\phi_{n}(y)\right) \circ \phi(x) \circ \exp \left(-\phi_{n}(y)\right) \chi_{\lambda}(t)^{-1}\right)(m) \\
& =\left(\chi_{\alpha}(t) \phi(x)\right)(m)
\end{aligned}
$$

where we use lemma 2.4.2 to conclude that $\exp \left(\phi_{n}(y)\right)$ and $\phi(x)$ commute. Thus

$$
\left(\rho(t) \circ \sigma(h) \circ \rho(t)^{-1}\right)(m)=\exp \left(\chi_{\alpha}(t) \phi(x)\right)(m)=\sigma\left(t h t^{-1}\right)(m)
$$

which proves the desired equation.

